NUFAC 05 Institute Lectures

Capri June 18/19 2005

R. B. Palmer (BNL)

2 Lectures and Tutorial

1. preface (inc Solenoid Focus)
2. Transverse Ionization Cooling
3. Longitudinal Ionization Cooling
4. Tutorials

On data stick:
05schoolv2.pdf, icoolman.pdf, & icool05.zip

On Web:
These Lectures and Tutorial
http://pubweb.bnl.gov/people/palmer/05school/05schoolv2.pdf
Files to Run problems with icool
http://pubweb.bnl.gov/people/palmer/05school/05icool.zip
Where to get generic icool files and manual
http://pubweb.bnl.gov/people/fernow/icool/readme.html

1 PREFACE

1.1 Units

When discussing the motion of particles in magnetic fields, I will use MKS units, but this means that momentum, energy, and mass are in Joules and kilograms, rather than in the familiar 'electron Volts'. To make the conversion easy, I will introduce these quantities in the forms: $[pc/e]$, $[E/e]$, and $[mc^2/e]$, respectively. Each of these expressions are then in units of straight Volts corresponding to the values of p, E and m expressed in electron Volts. For instance, I will write, for the bending radius in a field B:

$$\rho = \frac{[pc/e]}{B\ c}$$

meaning that the radius for a 3 GeV/c particle in 5 Tesla is

$$\rho = \frac{3\ 10^9}{5\ \times\ 3\ 10^8} = 2m$$

This units problem is often resolved in accelerator texts by expressing parameters in terms of $(B\rho)$ where this is a measure of momentum: the momentum that would have this value of $B \times \rho$, where

$$(B\rho) = \frac{[pc/e]}{c}$$

For 3 GeV/c, $(B\rho)$ is thus 10 (Tm), and the radius of bending in a field B=5 (T) is:

$$\rho = \frac{(B\rho)}{B} = \frac{10}{5} = 2m$$

1.2 Useful Relativistic Relations

$$
\begin{aligned}
dE &= \beta_v \, dp & (1)\\
\frac{dE}{E} &= \beta_v^2 \, \frac{dp}{p} & (2)\\
d\beta_v &= \frac{dp}{\gamma^2} & (3)
\end{aligned}
$$

I use β_v to denote v/c to distinguish it from the Courant-Schneider or Twiss parameters $\beta_\perp$

1.3 Emittance

$$\text{normalized emittance} = \frac{\text{Phase Space Area}}{\pi \text{ m c}}$$

The phase space can be transverse:p_x vs x, p_y vs y, or longitudinal Δp_z vs z, where Δp_z and z are with respect to the moving bunch center.

If x and p_x are both Gaussian and uncorrelated, then the area is that of an upright ellipse, and:

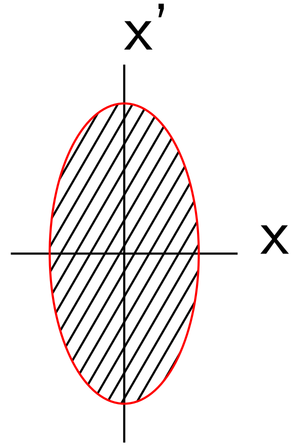
$$\epsilon_\perp = \frac{\pi \ \sigma_{p_\perp}\sigma_x}{\pi \ mc} = (\gamma\beta_v)\sigma_\theta\sigma_x \qquad (\pi \ m \ rad) \tag{4}$$

$$\epsilon_\parallel = \frac{\pi \ \sigma_{p_\parallel}\sigma_z}{\pi \ mc} = (\gamma\beta_v)\frac{\sigma_p}{p} \ \sigma_z \qquad (\pi \ m \ rad) \tag{5}$$

$$\epsilon_6 = \epsilon_\perp^2 \ \ \epsilon_\parallel \qquad (\pi \ m)^3 \tag{6}$$

Note that the π, added to the dimension, is a reminder that the emittance is phase space/π

1.4 Beta$_\perp$($Twiss$) of Beam



Upright phase ellipse in x' vs x,

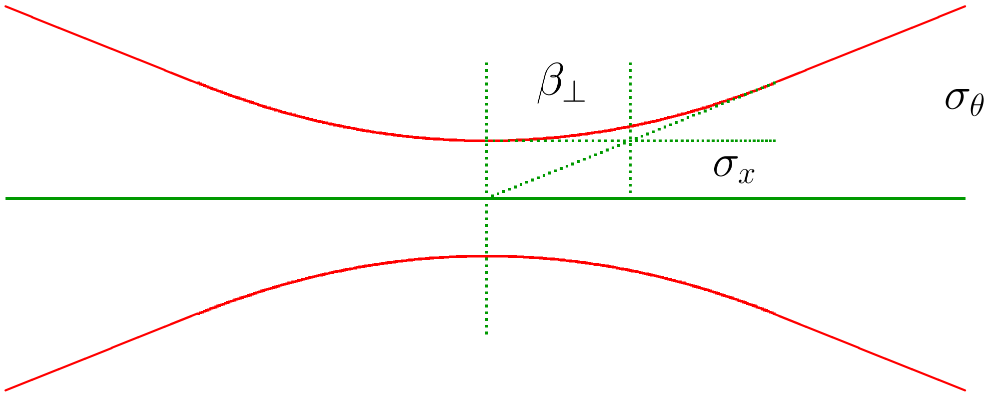
$$\beta_\perp \;=\; \left(\frac{\text{width}}{\text{height}} \quad \text{of phase ellipse}\right) \;=\; \frac{\sigma_x}{\sigma_\theta} \tag{7}$$

Then, using emittance definition:

$$\sigma_x \;=\sqrt{\epsilon_\perp\ \beta_\perp\ \frac{1}{\beta_v \gamma}} \tag{8}$$

$$\sigma_\theta \;=\sqrt{\frac{\epsilon_\perp}{\beta_\perp}\ \frac{1}{\beta_v \gamma}} \tag{9}$$

1.4.1 $\mathbf{Beta}_\perp (Twiss)$ **at focus**



$$\sigma_x = \sigma_o \sqrt{1 + \left(\frac{z}{\beta_\perp}\right)^2}$$

$\beta_\perp$ is like a depth of focus

As $z \rightarrow \infty$

$$\sigma_x \rightarrow \frac{\sigma_o\ z}{\beta_\perp}$$

giving an angular spread of

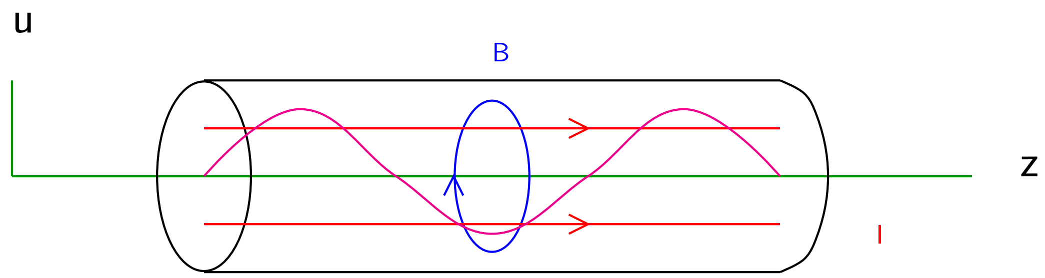
$$\theta = \frac{\sigma_o}{\beta_\perp}$$

as above in eq.7

1.4.2 Beta$_{\perp o}$($Courant - Schneider$) of a Lattice

$\beta_\perp$ above was defined by the beam, but a lattice can have a $\beta_{\perp o}$ that may or may not "match" a beam.

e,g. if continuous inward focusing force, as in a current carrying lithium cylinder (lithium lens), then there is a PERIODIC solution:



$$\frac{d^2u}{dz^2} = -k\ u \qquad u = A\ \sin\left(\frac{z}{\beta_{\perp o}}\right) \qquad u' = \frac{A}{\beta_o}\ \cos\left(\frac{z}{\beta_{\perp o}}\right)$$

where $k = cB/[pc/e]$ $\quad \beta_{\perp o} = 1/\sqrt{k} \quad \lambda = 2\pi\ \beta_o$

This particle motion is also an ellipse and

$$\frac{\text{width}}{\text{height}} \text{ of elliptical motion in phase space } = \frac{\hat{u}}{\hat{u'}} = \beta_{\perp o}$$

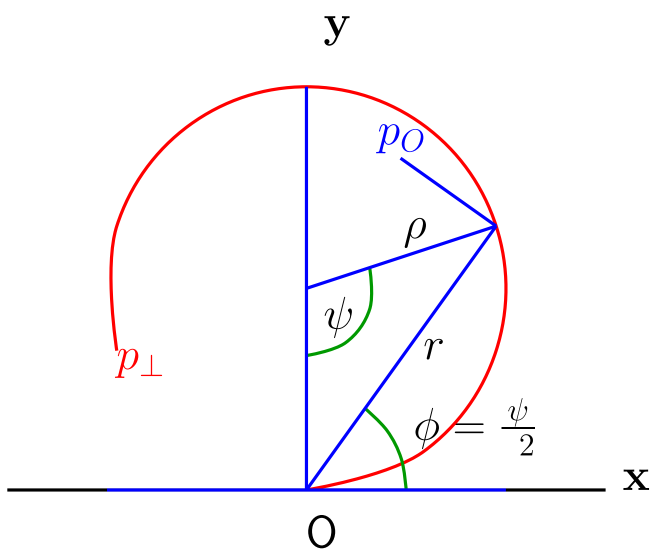
If we have many particles with$\beta_\perp(\text{Twiss}) = \beta_{\perp o}(\text{Courant Snyder})$ then all particles move arround the ellipse, and the shape, and thus $\beta_\perp(\text{Twiss})$ remains constant, and the beam is "matched" to this lattice.

If the beam's $\beta_\perp(\text{Twiss}) \neq \beta_{\perp \text{o}}$ of the system then $\beta_\perp(\text{Twiss}$ of the beam oscillates about $\beta_{\perp o}(\text{Courant Snyder})$: often refered to as a "beta beat".

1.5 Introduction to Solenoid Focussing

1.5.1 Motion in Long Solenoid

Consider motion in a fixed axial filed B_z, starting on the axis O with finate transverse momentum $p_\perp$ i.e. with initial angular momentum=0.



$$\rho = \frac{[pc/e]_\perp}{c\,B_z} \tag{10}$$

$$x = \rho \sin(\psi)$$

$$y = \rho\,(1 - \cos(\psi))$$

$$\frac{dz}{d\psi} = \rho\,\frac{p_z}{p_\perp}$$

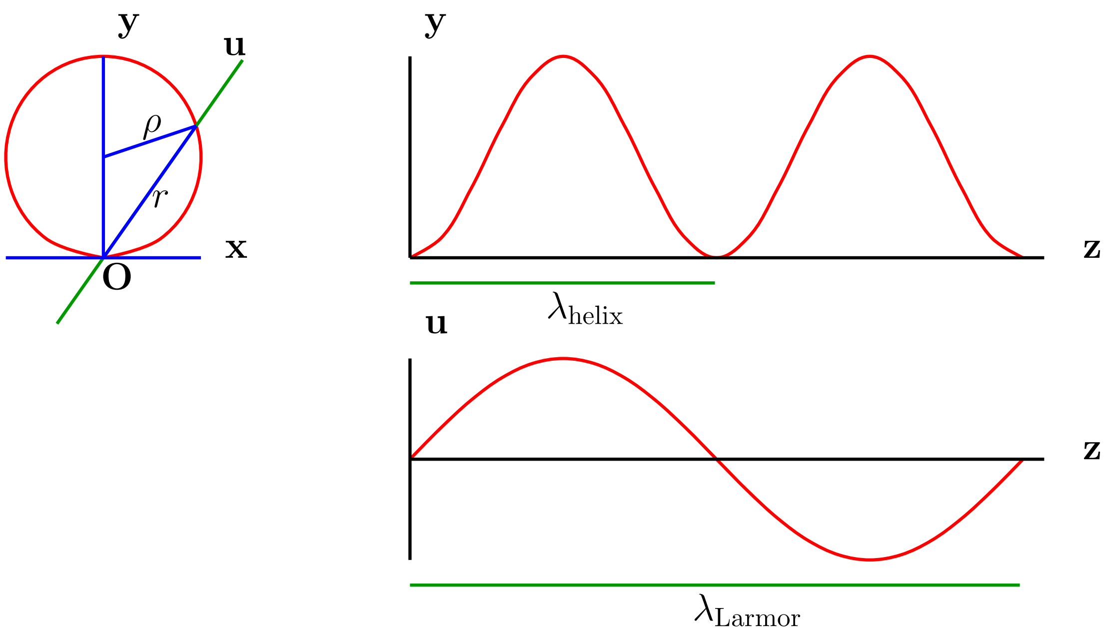
For $\psi < 180^o \quad \phi < 90^o$:

$$r = 2\rho\,\sin\left(\frac{\psi}{2}\right) = 2\rho \sin(\phi)$$

$$\frac{dz}{d\phi} = 2\,\rho\,\frac{p_z}{p_\perp}$$

1.5.2 Larmor Plane

If The center of the solenoid magnet is at O, then consider a plane that contains this axis and the particle. This, for a particle with initally no angular momentum, is the 'Larmor Plane:



$$u = 2\rho \ \sin(\phi) \tag{11}$$

$$\lambda_{\text{Helix}} = 2\pi \frac{dz}{d\psi} = 2\pi \ \rho \ \frac{p_z}{p_\perp} = 2\pi \ \frac{[pc/e]_z}{c \ B_z}$$

$$\lambda_{\text{Larmor}} = 2\pi \frac{dz}{d\phi} = 2\pi \ 2\rho \ \frac{p_z}{p_\perp} = 4\pi \ \frac{[pc/e]_z}{c \ B_z}$$

The lattice parameter β_o is defined in the Larmor frame, so

$$\beta_o = \frac{\lambda_{Larmor}}{2\pi} = \frac{2 \ [pc/e]_z}{c \ B_z} \tag{12}$$

1.5.3 Focusing Force

In this constant B case, the observed sinusoidal motion in the u plane is generated by a restoring force towards the axis O.

The momentum p_O about the axis O (perpendicular to the Larmor plane), using eq.10 and eq.11:

$$[p_O c/e] \;=\; [p_\perp c/e]\ \sin(\phi) \;=\; cB_z\rho\,\frac{u}{2\rho} \;=\; \frac{cB_z}{2}\,u \tag{13}$$

And the inward bending as this momentum crosses the B_z field is

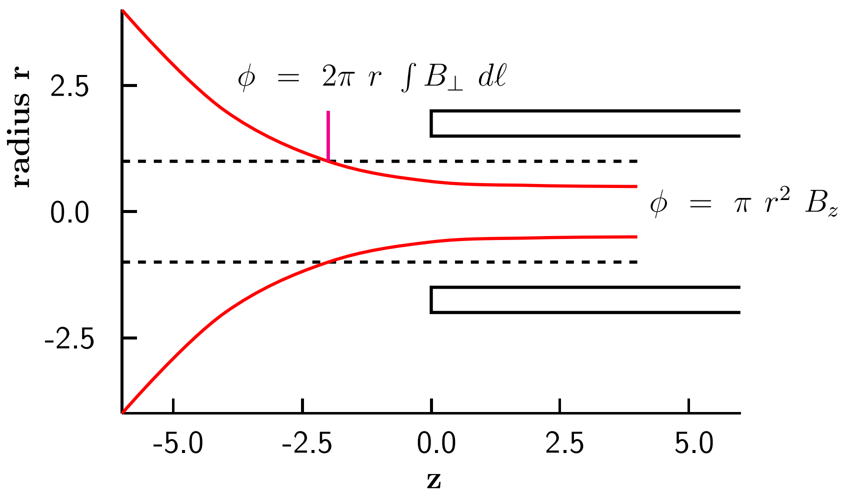
$$\frac{d^2u}{dz^2} \;=\; -\left(\frac{cB_z}{2\ [p_z c/e]}\right)^2\ u \tag{14}$$

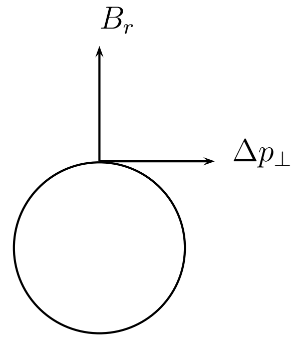
This inward force proportional to the distance u from the axis is an ideal focusing force

Note: the focusing "Force" $\propto\ B_z^2$ so it works the same for either sign, and $\propto\ 1/p_z^2$. Whereas in a quadrupole the force $\propto\ 1/p$ So solenoids are not good for high p, but beat quads at low p.

1.5.4 Entering a solenoid from outside

We will now look at a simple non-uniform B_z case. Let a particle start from the axis with finite transverse momentum, but no angular momentum. After some distance with no field, it reaches a radius u and then enters a solenoid with B_z. As it enters the solenoid it crosses radial field lines and receives some angular momentum.





$$\Delta[pc/e]_\perp \ = \ \int B_r \ dz \ = \ \frac{B_z \ r \ c}{2} \tag{15}$$

Sof for our case with zero initial transverse momentum,

$$[pc/e]_\perp \ = \ \int B_r \ dz \ = \ \frac{B_z \ r \ c}{2}$$

Which is the same as eq.13, and will lead to the same inward bending (eq.14), as when the particle started inside the field.

In fact eq.14 is true no matter how the axial field varies

1.5.5 Canonical Angular momentum

In general, for axial symmetry, a particle will have a conserved "Canonical Angular Momentum" $\mathcal{M}_o$ equal to the angular momentum outside the axial fields.

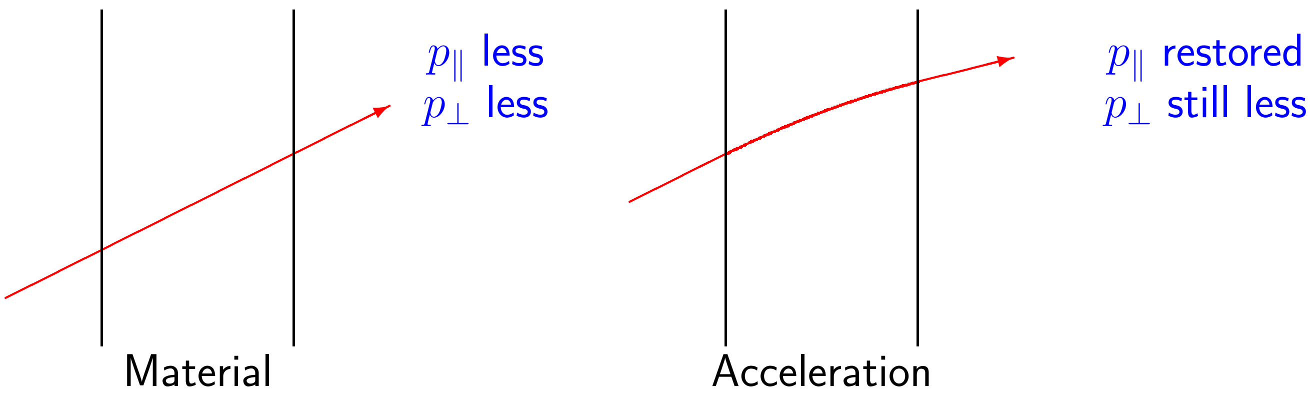
$$[\mathcal{M}c^2/e]_o \;=\; p_\perp\, r \text{ (Outdise the field)}$$

Inside a varying field $B_z(z)$, the real angular momentum will be:

$$[\mathcal{M}c^2/e] \;=\; [\mathcal{M}c^2/e]_o \;+\; \frac{r^2\; B_z\; c}{2}$$

But in the rotating Larmor Frame the angular momentum is always just the Canonical angular momentum, and motion in that frame has only inward focusing forces, with no angular kicks.

2 TRANSVERSE IONIZATION COOLING



2.1 Cooling rate vs. Energy

$$\text{(eq 4)} \quad \epsilon_{x,y} = \gamma\beta_v\, \sigma_\theta\, \sigma_{x,y}$$

If there is no Coulomb scattering, or other sources of emittance heating, then σ_θ and $\sigma_{x,y}$ are unchanged by energy loss, but p and thus $\beta\gamma$ are reduced. So the fractional cooling $d\epsilon\ /\epsilon$ is (using eq.2):

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} = \frac{dE}{E}\,\frac{1}{\beta_v^2} \tag{16}$$

which, for a given energy change, strongly favors cooling at low energy.

But if total acceleration were not important, e.g. if the cooling is done in a ring, then there is another criterion: The cooling per fractional loss of particles by decay:

$$Q = \frac{d\epsilon/\epsilon}{dn/n} = \frac{dp/p}{d\ell/c\beta_v\gamma\tau}$$

$$= \frac{dE/E \;\; 1/\beta_v^2}{d\ell/(c\gamma\beta_v\tau)}$$

$$= (c\tau/m_\mu) \;\; \frac{dE}{d\ell} \; \frac{1}{\beta_v}$$

Which only mildly favours low energy

2.2 Heating Terms

$$\epsilon_{x,y} = \gamma\beta_v \; \sigma_\theta \; \sigma_{x,y}$$

Between scatters the drift conserves emittance (Liouiville).
When there is scattering, $\sigma_{x,y}$ is conserved, but σ_θ is increased.

$$\Delta(\epsilon_{x,y})^2 = \gamma^2\beta_v^2 \; \sigma_{x,y}^2 \Delta(\sigma_\theta^2)$$

$$2\epsilon \; \Delta\epsilon = \gamma^2\beta_v^2 \left(\frac{\epsilon\beta_\perp}{\gamma\beta_v}\right) \; \Delta(\sigma_\theta^2)$$

$$\Delta\epsilon = \frac{\beta_\perp\gamma\beta_v}{2} \; \Delta(\sigma_\theta^2)$$

e.g. from Particle data booklet

$$\Delta(\sigma_\theta^2) \approx \left(\frac{14.1\ 10^6}{[pc/e]\beta_v}\right)^2\ \frac{\Delta s}{L_R}$$

$$\Delta\epsilon\ =\ \frac{\beta_\perp}{\gamma\beta_v^3}\,\Delta E\ \ \left(\left(\frac{14.1\ 10^6}{2[mc^2/e]_\mu}\right)^2\ \frac{1}{L_R dE/ds}\right)$$

Defining

$$C(mat,E)\ =\ \frac{1}{2}\ \left(\frac{14.1\ 10^6}{[mc^2/e]_\mu)}\right)^2\ \frac{1}{L_R\ d\gamma/ds} \tag{17}$$

then

$$\frac{\Delta\epsilon}{\epsilon}\ =\ dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ \ C(mat,E) \tag{18}$$

Equating this with equation 16

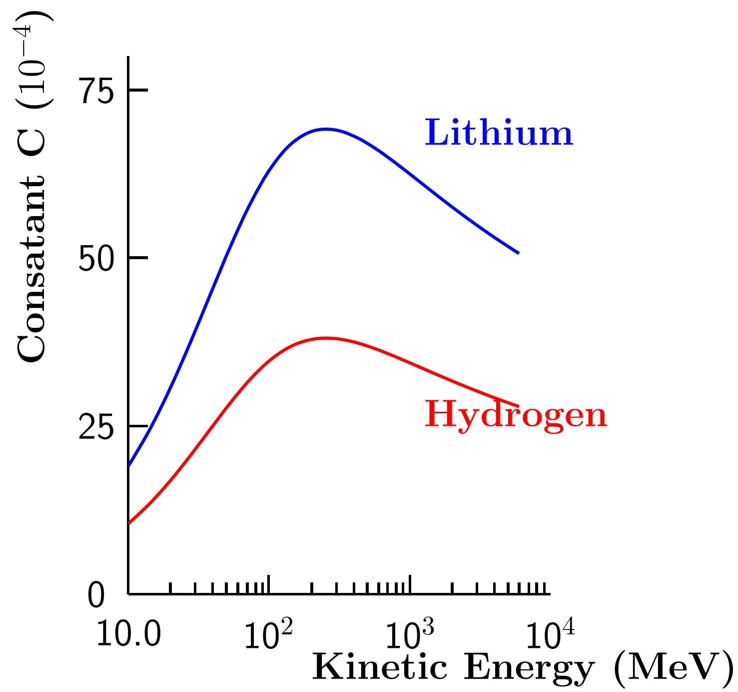
$$dE\,\frac{1}{\beta_v^2\ E}\ =\ dE\ \frac{\beta_\perp}{\epsilon\gamma\beta_v^3}\ \ C(mat,E)$$

gives the equilibrium emittance ϵ_o:

$$\epsilon_{x,y}(min)\ \ =\ \frac{\beta_\perp}{\beta_v}\ \ C(mat,E) \tag{19}$$

At energies such as to give minimum ionization loss, the constant C_o for various materials are approximately:

material	T	density	dE/dx	L_R	C_o
	^{o}K	kg/m^3	MeV/m	m	10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248



Clearly Liquid Hydrogen is far the best material, but has cryogenic and safety complications, and requires windows made of Aluminum or other material which will significantly degrade the performance.

2.3 Rate of Cooling

$$\frac{d\epsilon}{\epsilon} = \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right) \frac{dp}{p} \tag{20}$$

2.4 Beam Divergence Angles

$$\sigma_\theta \;=\; \sqrt{\frac{\epsilon_\perp}{\beta_\perp\;\beta_v\gamma}}$$

so, from equation 19, for a beam in equilibrium

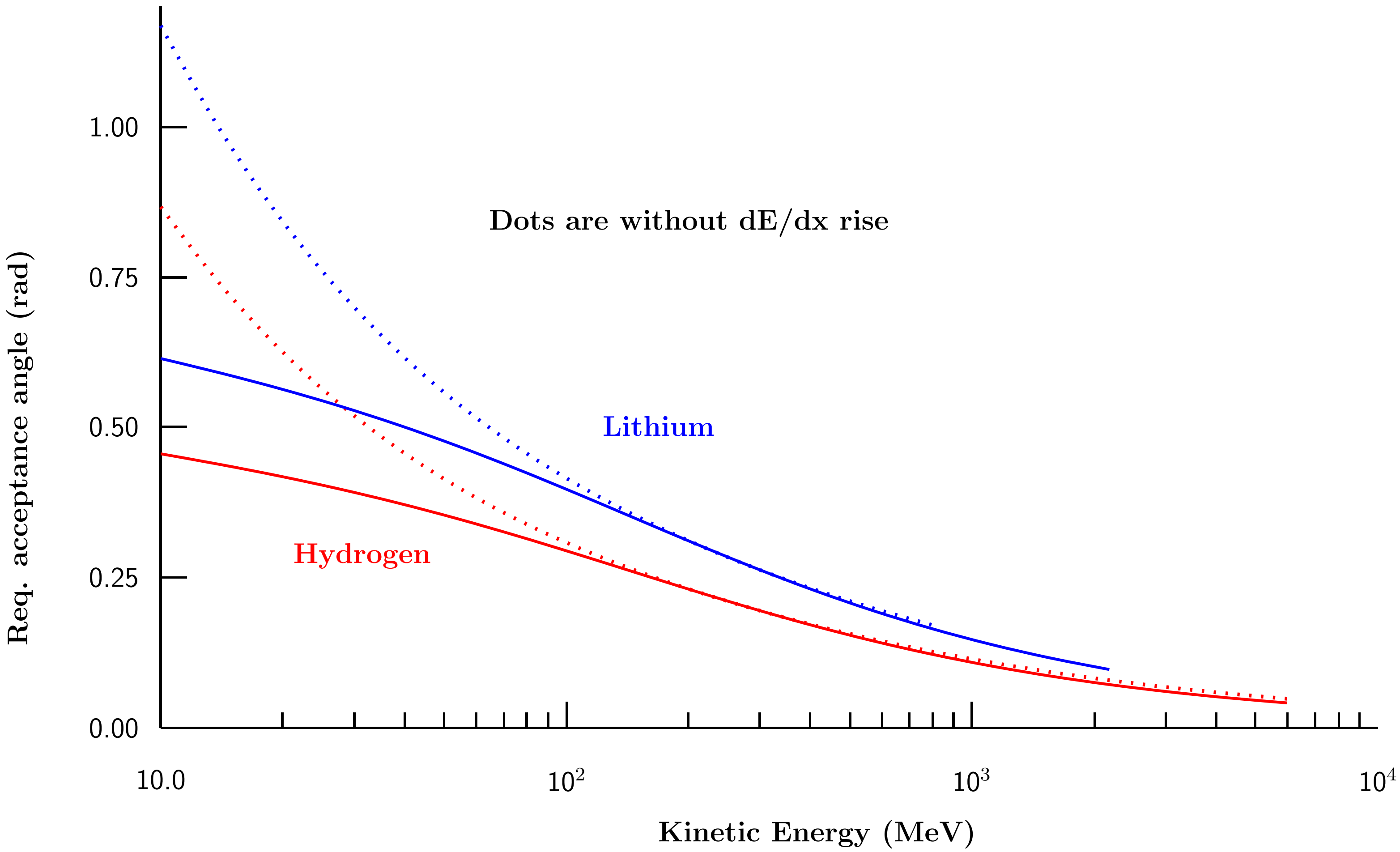
$$\sigma_\theta \;=\; \sqrt{\frac{C(mat,E)}{\beta_v^2\gamma}}$$

and for 50 % of maximum cooling rate and an aperture at 3 σ, the angular aperture $\mathcal{A}$ of the system must be

$$\mathcal{A} \;=\; 3\sqrt{2}\,\sqrt{\frac{C(mat,E)}{\beta_v^2\gamma}} \tag{21}$$

Apertures for hydrogen and lithium are plotted vs. energy below. These are very large angles, and if we limit apertures to less than 0.3, then this requirement sets lower energy limits of about 100 MeV ($\approx$ 170 MeV/c) for Lithium, and about 25 MeV ($\approx$ 75 MeV/c) for hydrogen.

$\theta = 0.3$ may be about as large as is possible in a lattice, but larger angles may be sustainable in a continuous focusing system such as a lens or solenoid. is optimistic, as we will see in the tutorial.


Dots are without dE/dx rise
Lithium
Hydrogen
Req. acceptance angle (rad)
1.00
0.75
0.50
0.25
0.00
10.0
10^2
10^3
10^4
Kinetic Energy (MeV)

2.5 Focusing Systems

2.5.1 Solenoid

In a solenoid with axial field B_{sol} (from eq 12)

$$\beta_\perp = \frac{2\,[pc/e]}{c\,B_{sol}}$$

so

$$\epsilon_{x,y}(min) = C(mat,E)\ \frac{2\,\gamma\,[mc^2/e]_\mu}{B_{sol}\ c} \tag{22}$$

For $E = 100\ MeV$ ($p \approx 170\ MeV/c$), $B = 10\ T$, then $\beta \approx 11.4\ cm$. and

$\epsilon_{x,y}(min) \approx 532(\pi\ mm\ mrad)$.

2.5.2 Current Carrying Rod

In a rod carrying a uniform axial current, the azimuthal magnetic field B varies linearly with the radius r. A muon traveling down it is focused:

$$\frac{d^2r}{dr^2} = -\frac{B\ c}{[pc/e]} = -\left(\frac{c}{[pc/e]}\ \frac{dB}{dr}\right)\ r$$

so orbits oscillate with

$$\beta_\perp^2 = \frac{\gamma\beta_v}{dB/dr}\ \frac{[mc^2/e]_\mu}{c} \tag{23}$$

If we set the rod radius a to be f_{ap} times the rms beam size $\sigma_{x,y}$ (from eq.8),

$$\sigma_{x,y} = \sqrt{\frac{\epsilon_{x,y}\ \beta_\perp}{\beta_v\gamma}}$$

and if the field at the surface is B_{max}, then

$$\beta_\perp^2 = \frac{\gamma\beta_v[mc^2/e]_\mu\ f_{ap}}{B_{max}\ c}\ \sqrt{\frac{\epsilon_{x,y}\ \beta}{\gamma\ \beta_v}}$$

from which we get:

$$\beta_\perp = \left(\frac{f_{ap}\ [mc^2/e]_\mu}{B_{max}\ c}\right)^{2/3}\ (\gamma\beta_v\ \epsilon_{x,y})^{1/3}$$

puting this in equation 19

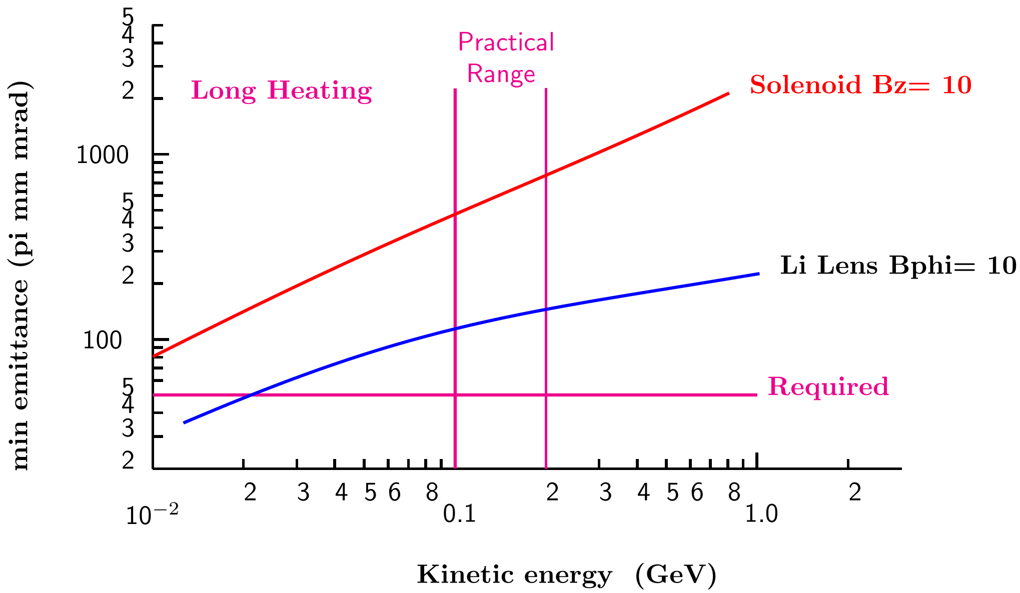
$$\epsilon_{x,y}(min) = (C(mat,E))^{1.5}\left(\frac{f_{ap}\ [mc^2/e]_\mu}{B_{max}\ c\ \beta_v}\right)\ \sqrt{\gamma} \tag{24}$$

e.g. B_{max}=10 T, f_{ap}=3, E=100 MeV, then $\beta_\perp = 1.23$ cm, and $\epsilon_{x,y}(min)$=100 (π mm mrad)

The choice of a maximum surface field of 10 T is set by breaking of the containing pipe in current solid Li designs. With liquid Li a higher field may be possible.

2.5.3 Compare Focusing

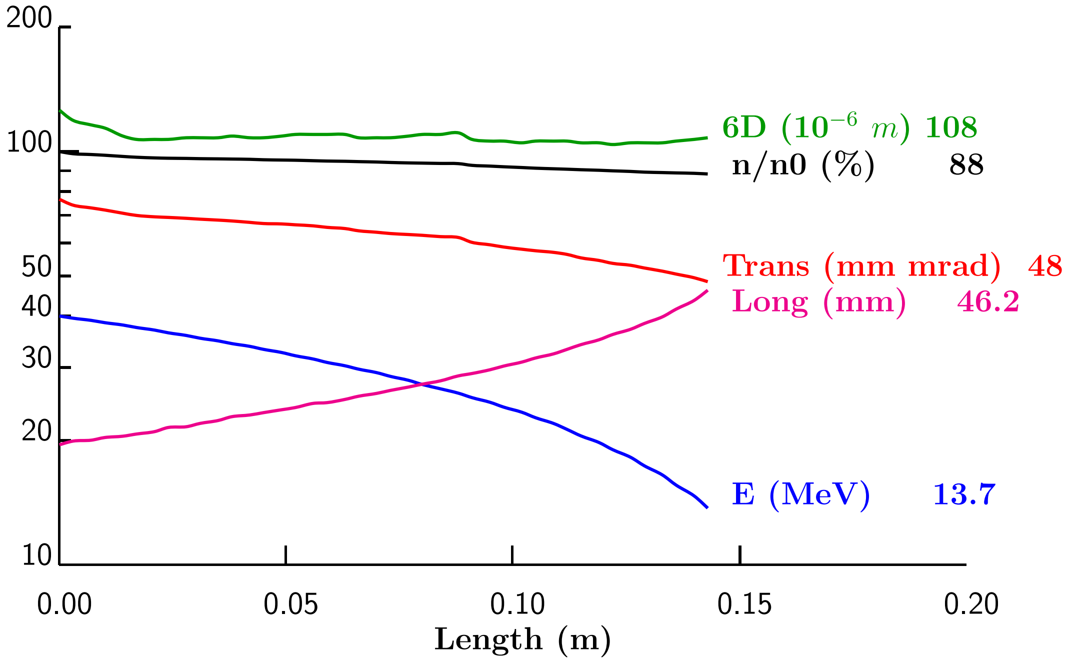
Comparing the methods as a function of the beam kinetic energy.

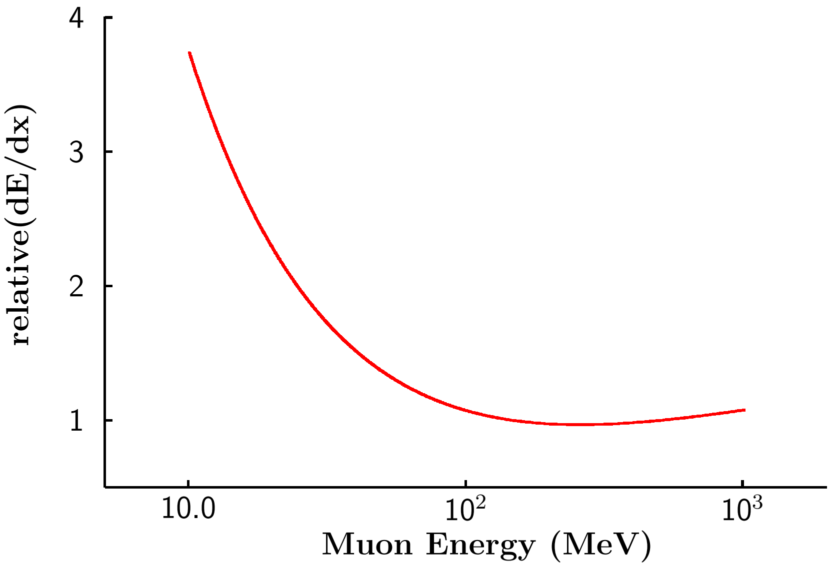


We see that, for the parameters selected, The lithium rod achieves a lower emittance than the solenoid despite its higher C value. Neither method allows transverse cooling below about 80 (π mm mrad), except at very low energies.

2.6 Li Lens to meet 3 TeV Collider Final Emittance

At lower energies the minimum emittance is lower and can meet the 50 pi mm mrad requirement for a 5 TeV collider. But the dp/p rises rapidly because of the reverse slope of dE/dx. Never the less, the 6D emittance does not rise. The effect is that of an emittance exchange between longitudinal and transverse emittances.

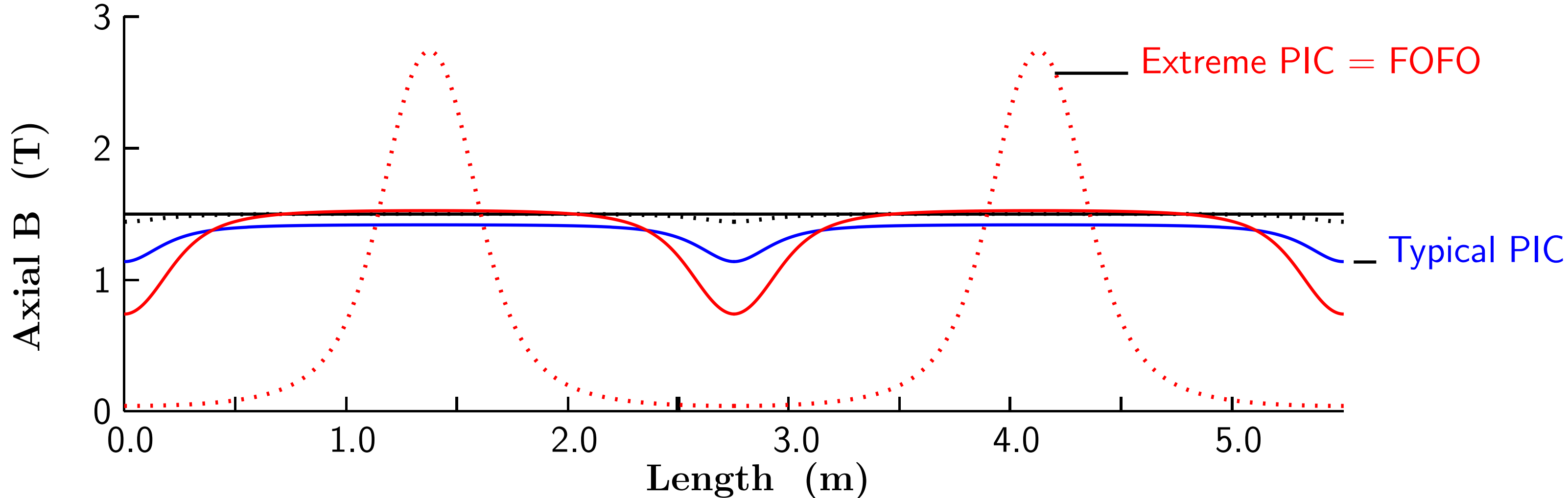


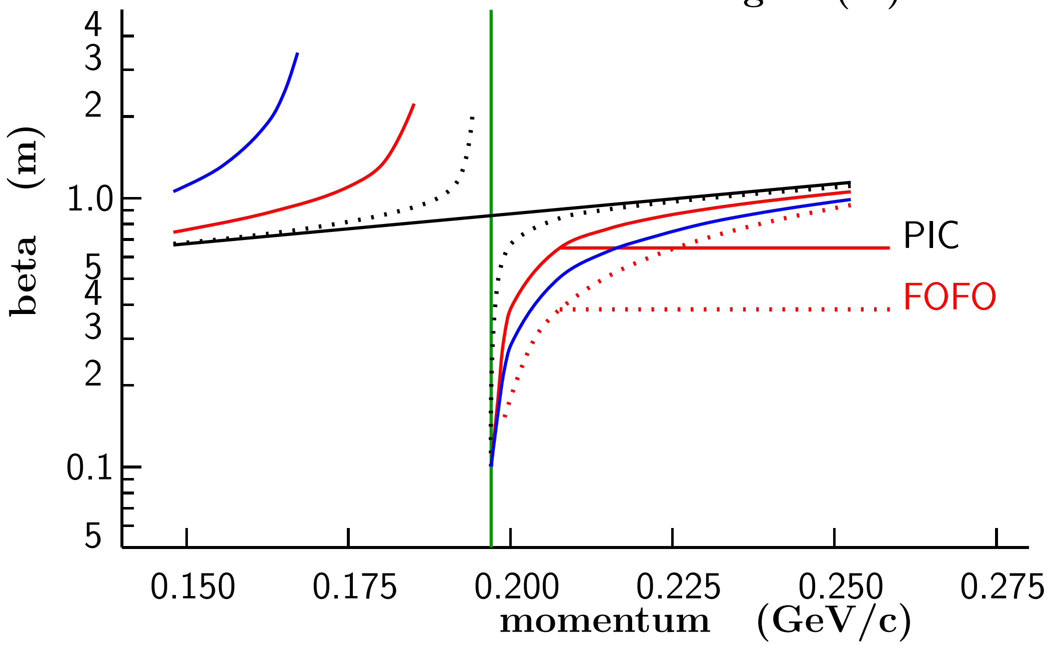


- Work needed on Matching in and out

2.7 Decreasing beta in Solenoids by adding periodicity

Parametric-Resonance Ionization Cooling (PIC) (Derbenev[1]) / FOFO (Palmer)



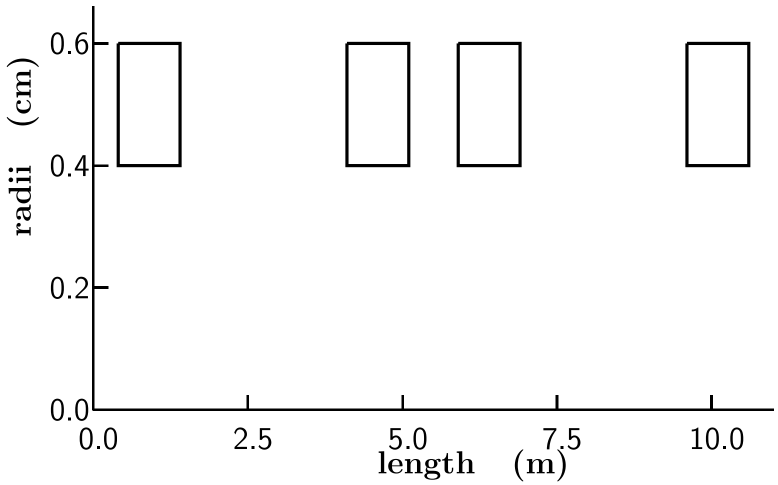


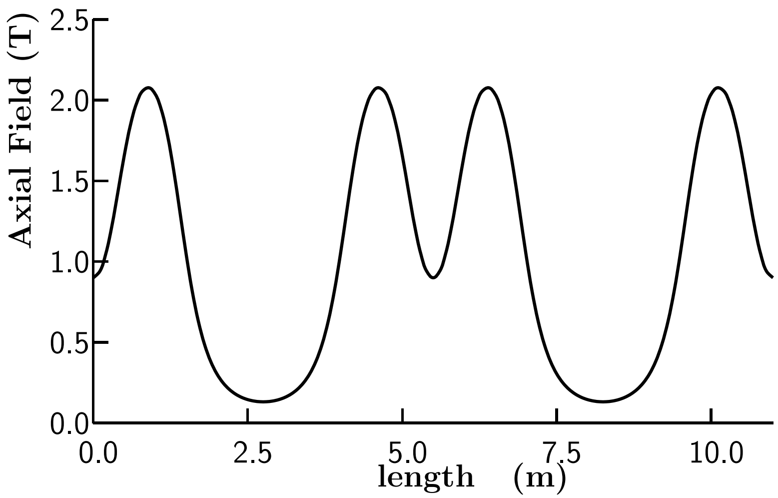
- Determination of lattice betas
  - Track single near paraxial particle through many cells
  - plot θ_x vs x after each cell
  - fit ellipse: $\beta_{x,y}$ = A((x) / A(θ_x)
- Resonances introduced
- Betas reduced locally
- Momentum acceptance small

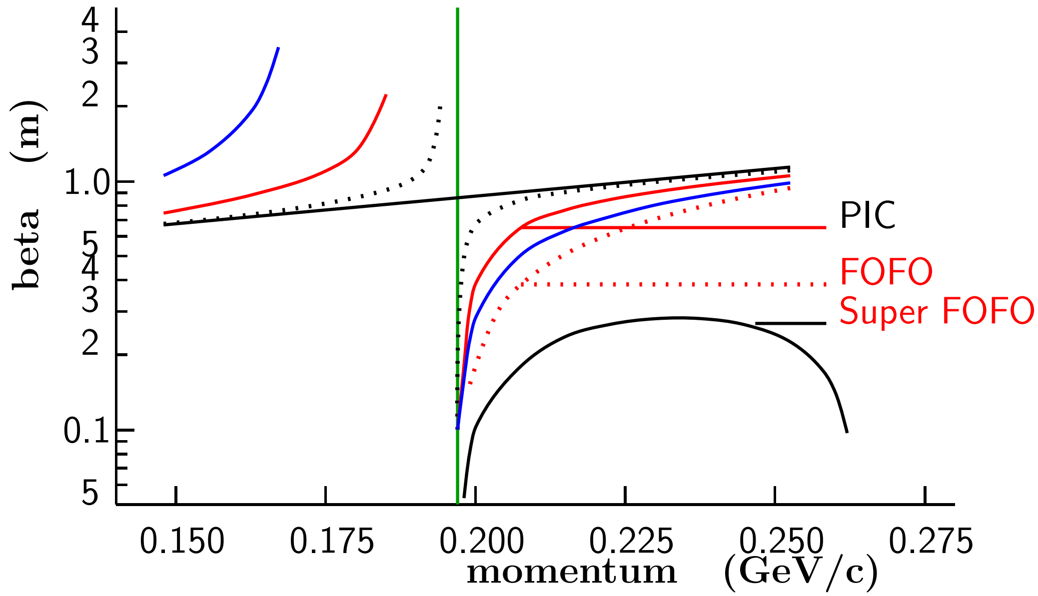
[1] http://www.fnal.gov/projects/muon_collider/FridayMeetings/May%2013%2C%202005/Simulations%20of%20parametric%20Resonance%20Cooling%20-%20Beard.pdf

Super FOFO

Double periodicity







- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

2.8 Angular Momentum Problem in Solenoid Cases

In the absence of external fields and energy loss in materials, the angular momentum of a particle is conserved.

But a particle entering a solenoidal field will cross radial field components and its angular momentum ($r\ p_\phi$) will change (eq.15).

$$r\ \Delta([pc/e]_\phi)\ =\ r\ \Delta\left(\frac{c\ B_z\ r}{2}\right)$$

If, in the absence of the field, the particle had "canonical" angular momentum $(p_\phi\ r)_{\rm can}$, then in the field it will have angular momentum:

$$[pc/e]_\phi\ r\ =\ (p_\phi\ r)_{\rm can}\ +\ \left(\frac{c\ B_z\ r}{2}\right)\ r$$

so

$$[pc/e]_\phi\ r)_{\rm can}\ =\ [pc/e]_\phi\ r\ -\ \left(\frac{c\ B_z\ r}{2}\right)\ r \tag{25}$$

If the initial average canonical angular momentum is zero, then in B_z:

$$<[pc/e]_\phi\ r> = \left(\frac{c\ B_z\ r}{2}\right)\ r$$

Material introduced to cool the beam, will reduce all momenta, both longitudinal and transverse, random and average.

Re-acceleration will not change the angular momenta, so the average angular momentum will continuously fall.

Consider the case of almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis.

$$[pc/e]_\phi \, r \;\approx\; 0$$

and there is now a finite average canonical momentum (from eq.25):

$$< [pc/e]_\phi \, r >_{\rm can} = -\left(\frac{c\; B_z\; r}{2}\right)\; r$$

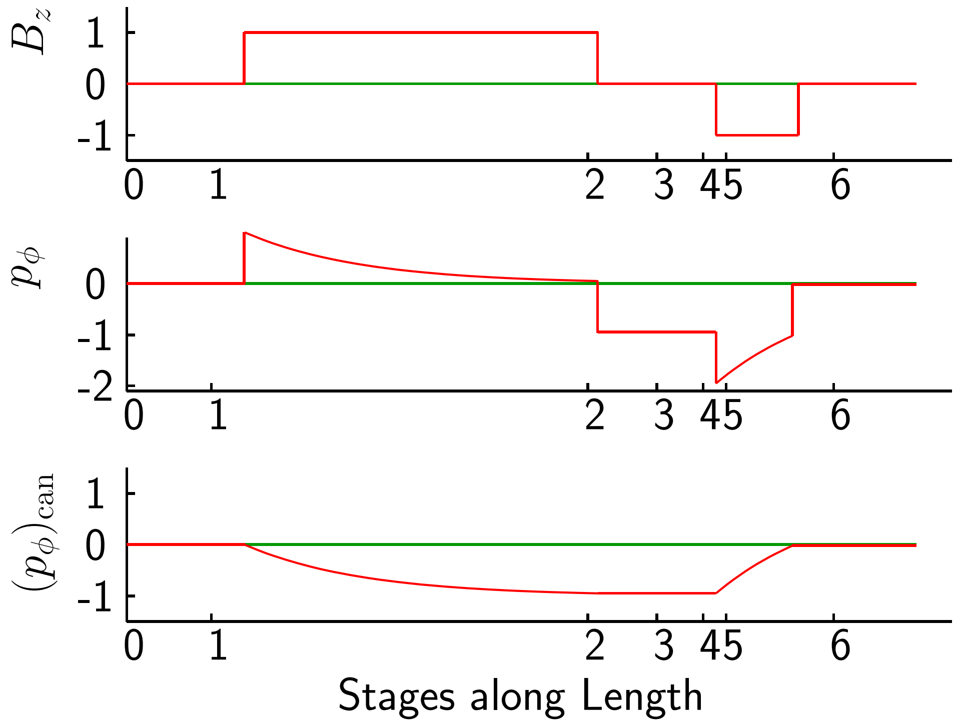
When the beam exits the solenoid, then this canonical angular momentum becomes a real angular momentum and represents an effective emittance, and severely limits the possible cooling.

$$< [pc/e]_\phi \, r >_{\rm end} = \; -\left(\frac{c\; B_z\; r}{2}\right)\; r$$

The only reasonable solution is to reverse the field, either once, a few, or many times.

2.8.1 Single Field Reversal Method

The minimum required number of field "flips" is one.



After exiting the first solenoid, we have real coherent angular momentum:

$$([pc/e]_\phi\ r)_3\ =\ -\left(\frac{c\ B_{z1}\ r}{2}\right)\ r$$

The beam now enters a solenoid with opposite field $B_{z2} = -B_{z1}$.
The canonical angular momentum remains the same, but the real angular momentum is dou-

bled.

$$([pc/e]_\phi \; r)_4 \;=\; -2\left(\frac{c\; B_{z1}\; r}{2}\right)\; r$$

We now introduce enough material to halve the transverse field components. Then

$$([pc/e]_\phi \; r)_5 \;=\; -\left(\frac{c\; B_{z1}\; r}{2}\right)\; r$$

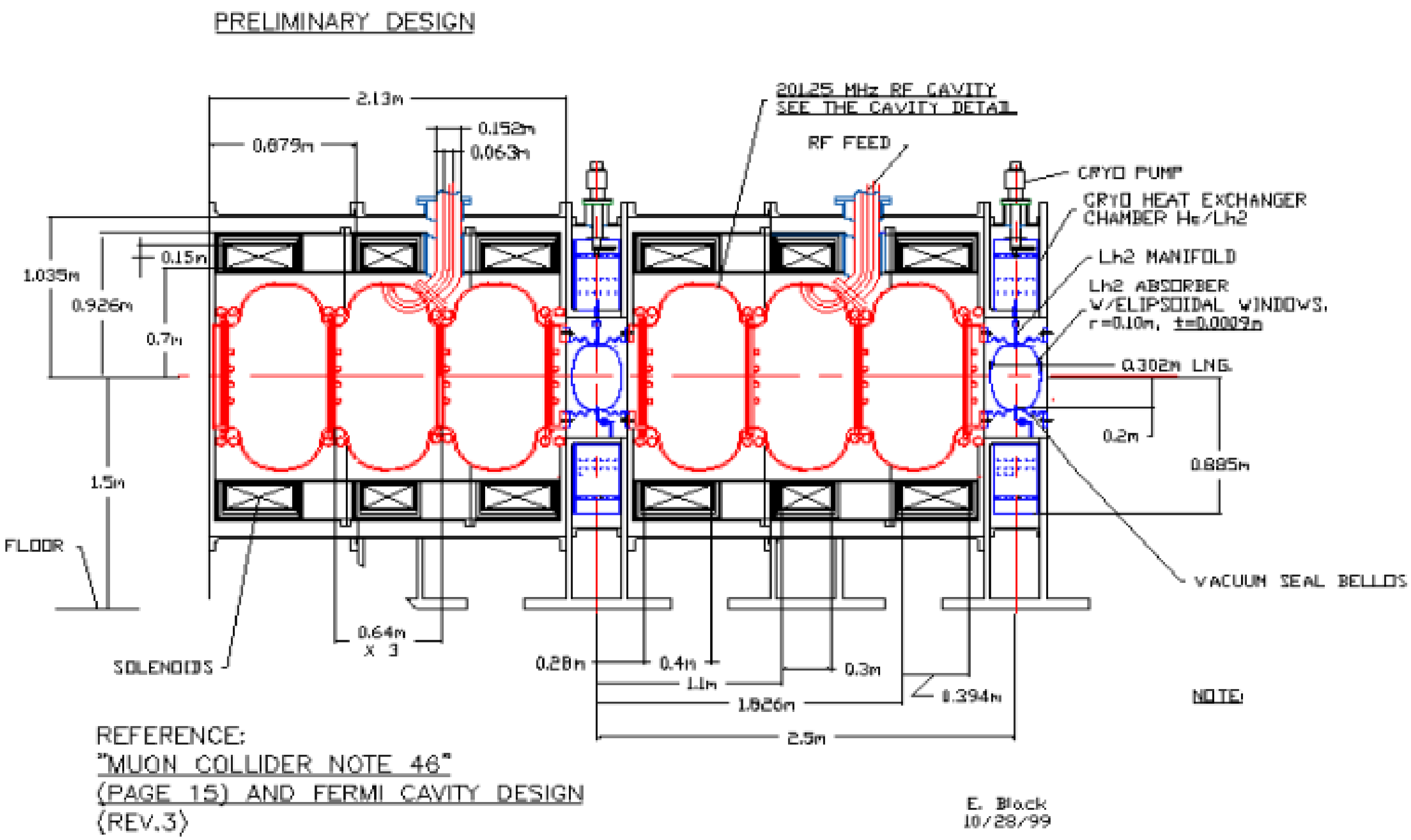
This is inside the field $B_{z2} \;=\; -B_{z1}$. The canonical momentum, and thus the angular momentum on exiting, is now:

$$([pc/e]_\phi \; r)_6 \;=\; -\left(\frac{c\; B_{z1}\; r}{2}\right)\; r \;-\; -\left(\frac{c\; B_{z1}\; r}{2}\right)\; r \;=\; 0$$

2.9 Examples of Focus Design

2.9.1 Continuous Solenoid

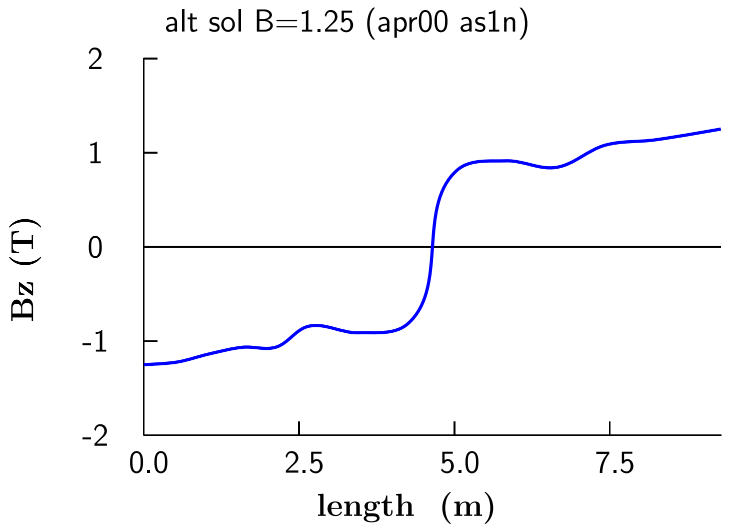
Coils Outside RF: e.g. FNAL 1 flip

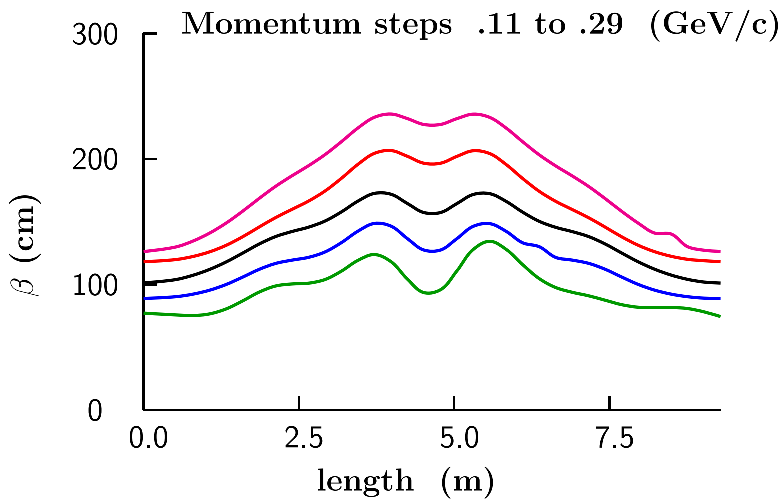


"Flip" Design

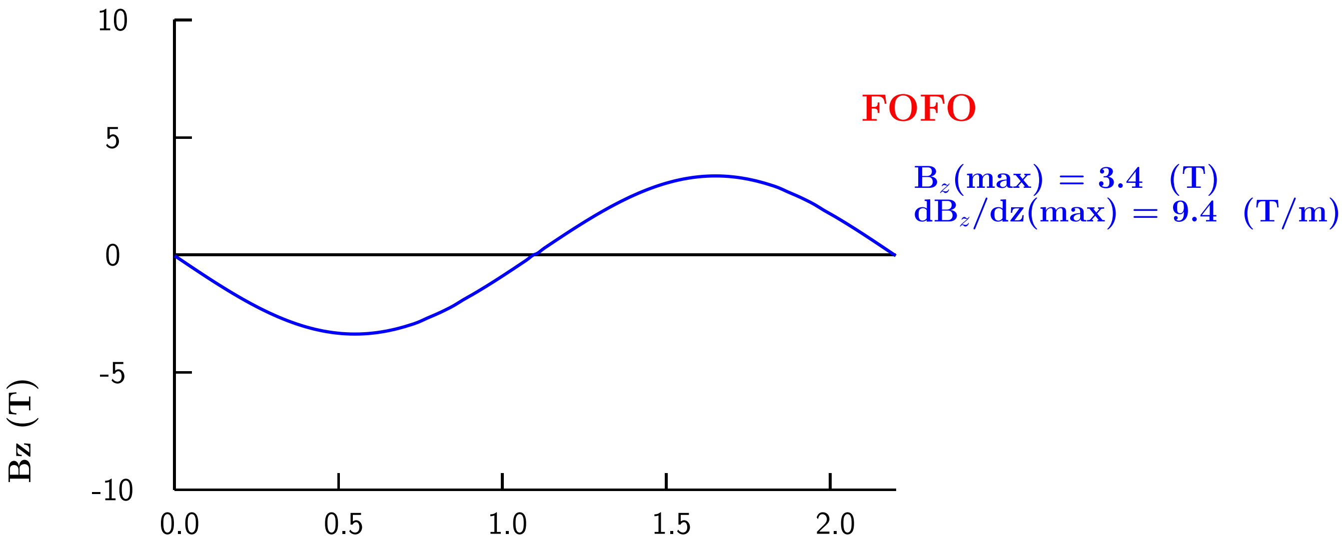
One must design the flips to match the betas from one side to the other.

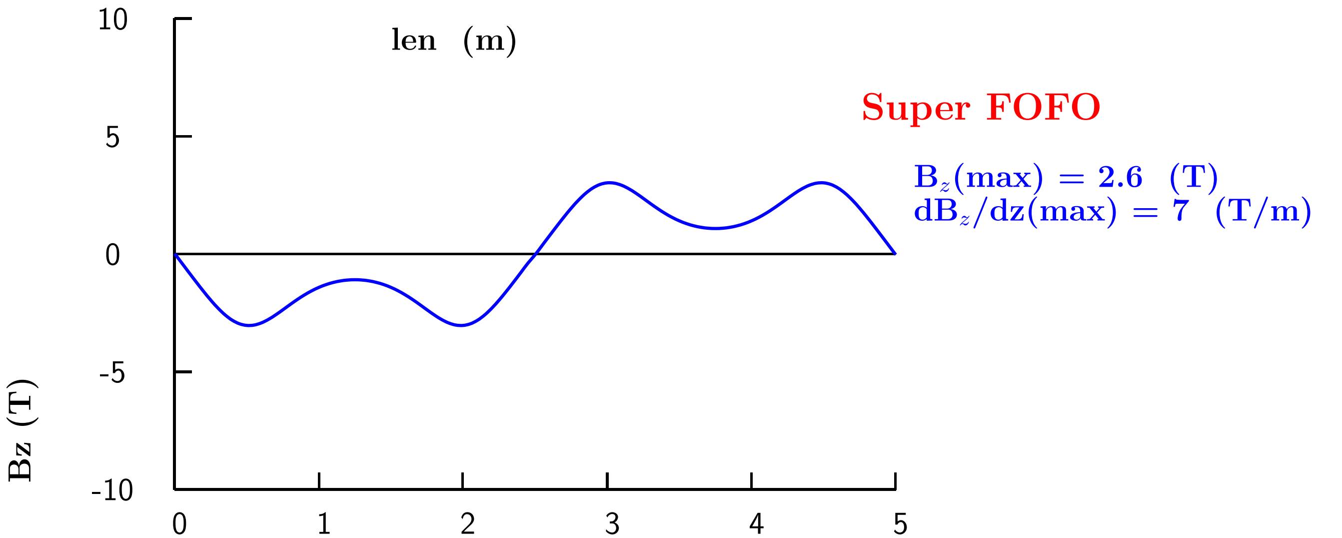
For a computer designed matched flip between uniform solenoidal fields: the following figure shows B_z vs. z and the $\beta_\perp$'s vs. z for different momenta.



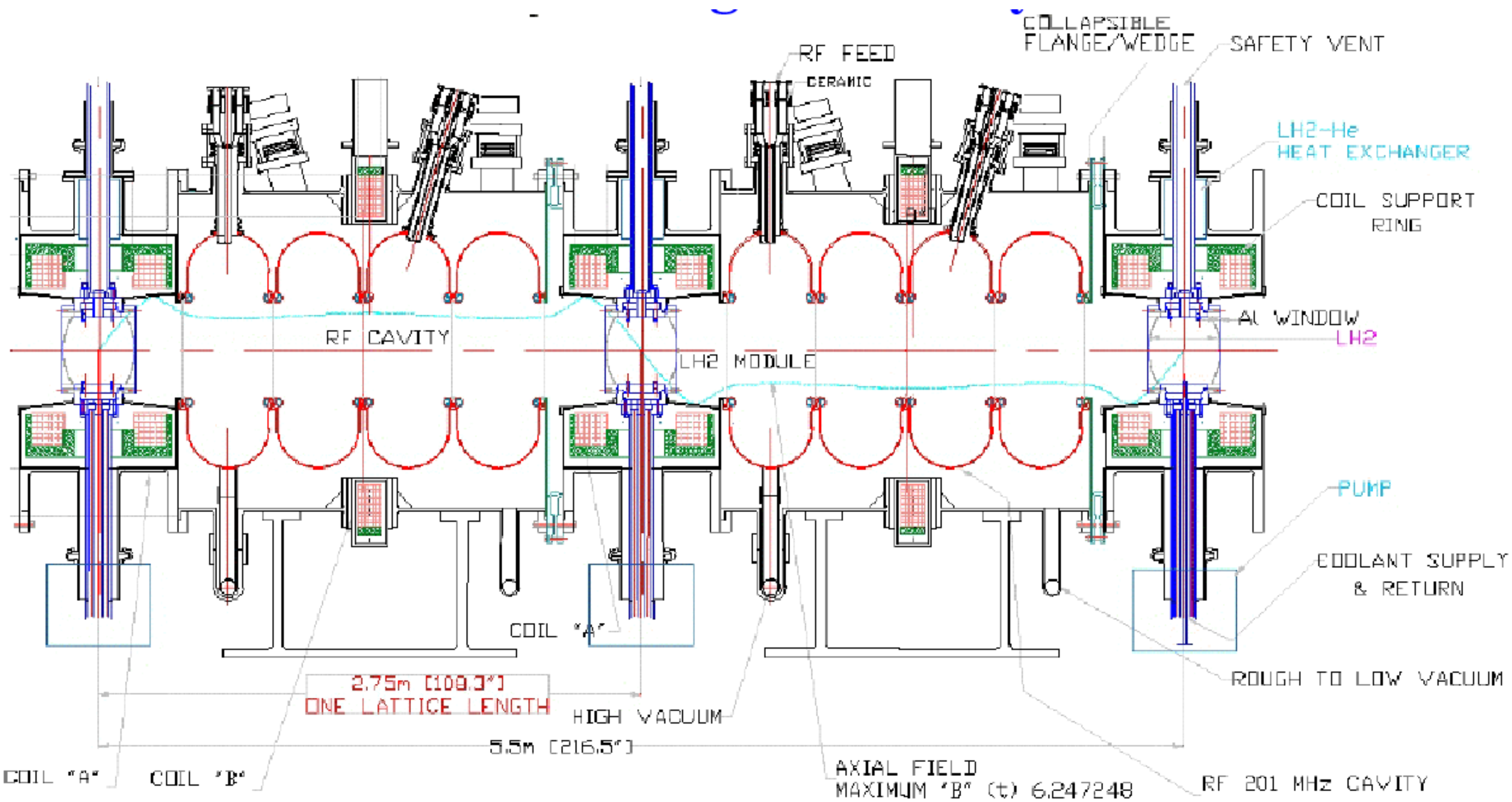


2.9.2 Lattices with many "flips"



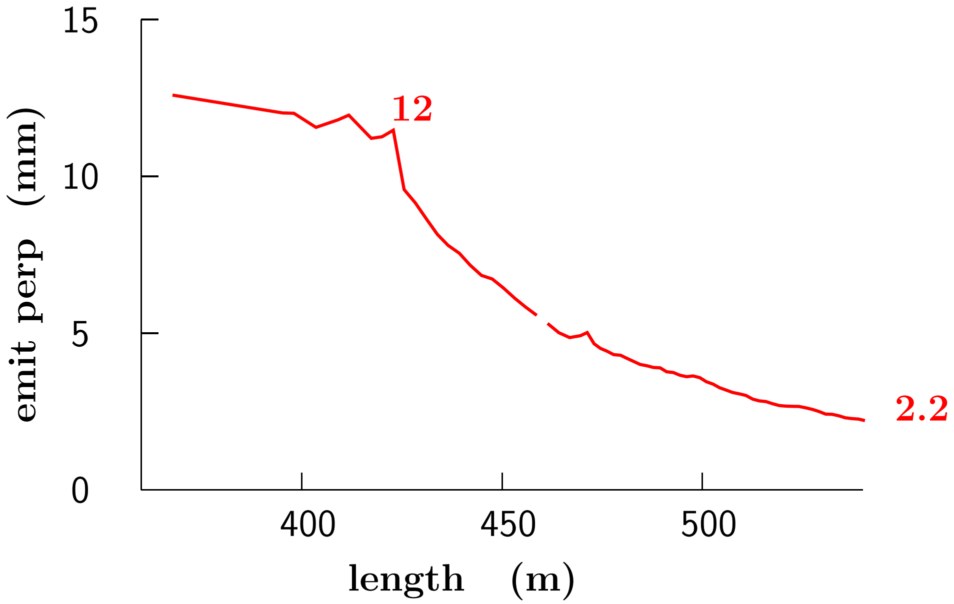


SFOFO Lattice Engineering
Study 2 at Start of Cooling



- This is the lattice to be tested in Muon Ionization Cooling Experiemnt (MICE) at RAL
- In study 2 the lattice is modified vs. length to lower $\beta_\perp$ as ϵ falls
  This keeps σ_θ and ϵ/ϵ_o more or less constant, thus maintains cooling rate

Study 2 Performance



With RF and Hydrogen Windows, $C_o \approx 45\ 10^{-4}$
$\beta_\perp$(end)=.18 m, β_v(end)=0.85, So

$$\epsilon_\perp(\mathrm{min}) = \frac{45\ 10^{-4}\ 0.18}{0.85} = 0.95 \ \ (\pi mm\ mrad)$$

$$\frac{\epsilon_\perp}{\epsilon_\perp(\mathrm{min})} \approx 2.3$$

so from eq. 20

$$\frac{d\epsilon}{\epsilon}(\mathrm{end}) = \left(1 - \frac{\epsilon}{\epsilon(\mathrm{min})}\right) \frac{dp}{p} \approx 0.57 \frac{dp}{p}$$

3 LONGITUDINAL IONIZATION COOLING

Following the convention for synchrotron cooling we define partition functions:

$$J_{x,y,z} = \frac{\frac{\Delta\ (\epsilon_{x,y,z})}{\epsilon_{x,y,z}}}{\frac{\Delta p}{p}} \tag{26}$$

$$J_6 = J_x + J_y + J_z \tag{27}$$

where the $\Delta\epsilon$'s are those induced directly by the energy loss mechanism (ionization energy loss in this case). Δp and p refer to the loss of momentum induced by this energy loss.

In synchrotrons, with no gradients fields, $J_x = J_y = 1$, and $J_z = 2$.

In ionization cooling, $J_x = J_y = 1$, but J_z is negative or small.

3.1 c.f. Transverse

From last lecture:

$$\frac{\Delta\sigma_{p\perp}}{\sigma_{p\perp}} = \frac{\Delta p}{p}$$

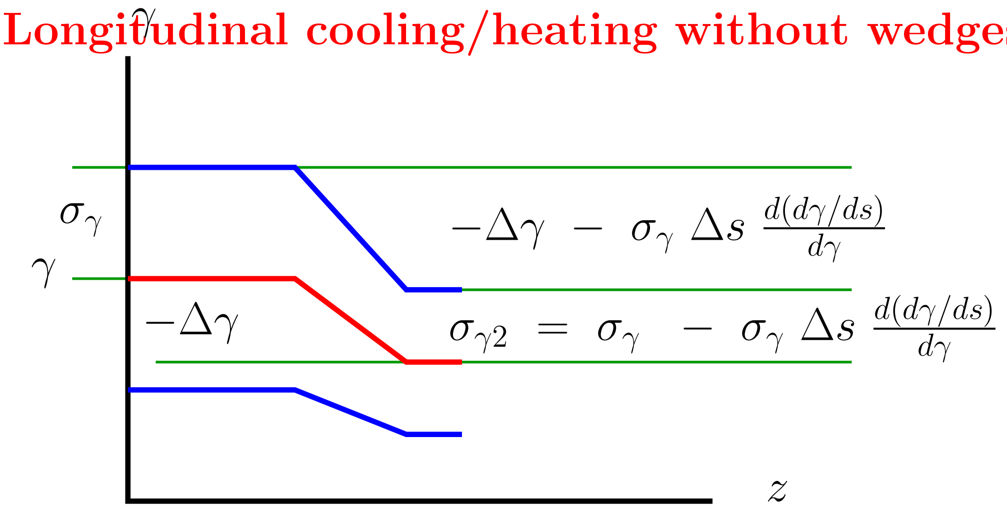
and $\sigma_{x,y}$ does not change, so

$$\frac{\Delta\epsilon_{x,y}}{\epsilon_{x,y}} = \frac{\Delta p}{p} \tag{28}$$

and thus

$$J_x = J_y = 1 \tag{29}$$

3.2 Longitudinal cooling/heating without wedges



The emittance in the longitudinal direction ϵ_z is (eq.5):

$$\epsilon_z \;=\; \gamma\beta_v\,\frac{\sigma_p}{p}\,\sigma_z \;=\; \frac{1}{m}\,\sigma_p\sigma_z \;=\; \frac{1}{m}\,\sigma_E\sigma_t \;=\; c\,\sigma_\gamma\,\sigma_t$$

where σ_t is the rms bunch length in time, and c is the velocity of light. Drifting between interactions will not change emittance (Louville), and an interaction will not change σ_t, so emittance change is only induced by the energy change in the interactions:

For a wedge with center thickness ℓ and height from center h ($2h\ \tan(\theta/2)=\ell$), in dispersion D ($D=\frac{dy}{dp/p}$, or with eq.2: $D=\beta_v^2\,\frac{dy}{d\gamma/\gamma}$) (see fig. above):

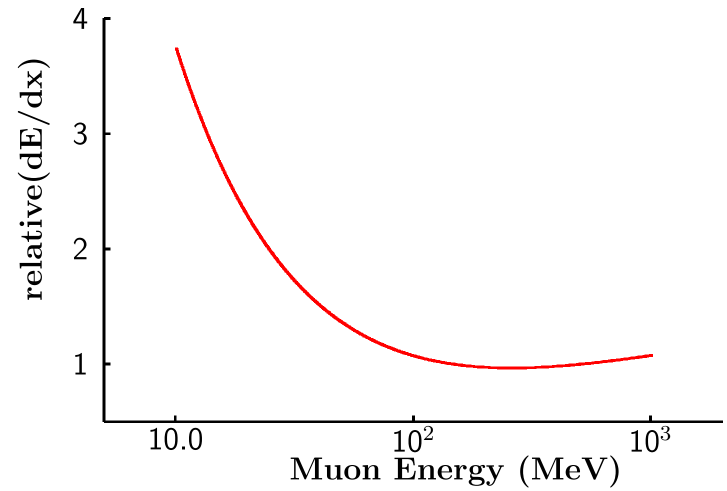
$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \;=\; \frac{\sigma_\gamma\;\Delta s\;\frac{d(d\gamma/ds)}{d\gamma}}{\sigma_\gamma} \;=\; \Delta s\;\frac{d(d\gamma/ds)}{d\gamma}$$

and

$$\frac{\Delta p}{p} \;=\; \frac{\Delta\gamma}{\beta_v^2\gamma} \;=\; \frac{\ell}{\beta_v^2\gamma}\;\left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$J_z = \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} = \frac{\left(\Delta s \, \frac{d(d\gamma/ds)}{d\gamma}\right)}{\frac{\Delta s}{\beta_v^2\gamma} \, \left(\frac{d\gamma}{ds}\right)} = \frac{\left(\beta_v^2 \, \gamma \frac{d(d\gamma/ds)}{d\gamma}\right)}{\left(\frac{d\gamma}{ds}\right)} \tag{30}$$



A typical relative energy loss as a function of energy is shown above (this example is for Lithium). It is given approximately by:

$$\frac{d\gamma}{ds} = B \, \frac{1}{\beta_v^2} \, \left(\frac{1}{2} \, ln(A \, \beta_v^4\gamma^4 \; - \; \beta_v^2\right) \tag{31}$$

where

$$A = \frac{(2m_e c^2/e)^2}{I^2} \qquad B \approx \frac{0.0307}{(m_\mu c^2/e)} \, \frac{Z}{A} \tag{32}$$

where Z and A are for the nucleus of the material, and I is the ionization potential for that material.

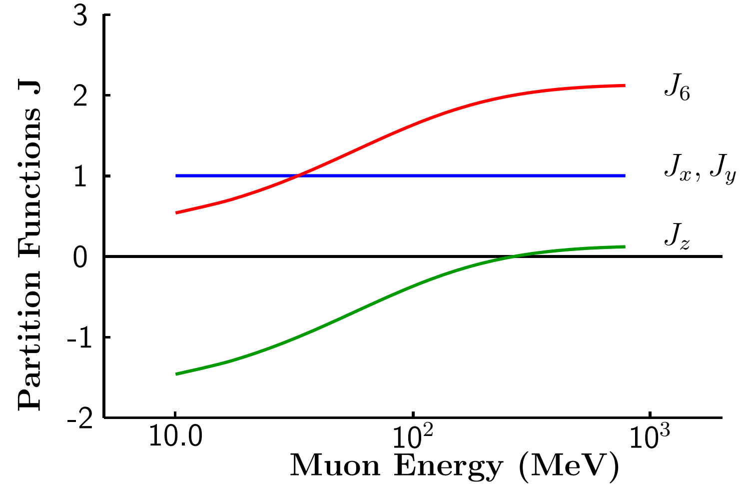
Differentiating the above:

$$\frac{\delta(d\gamma/ds)}{\delta\gamma} = \frac{B}{\beta_v} \left(\frac{2}{\beta_v\gamma} - \frac{1}{(\beta_v\gamma)^3} \, ln(A \, \beta_v^4\gamma^4) \; + \; \frac{2}{(\beta_v\gamma)^3}\right)$$

Substituting this into equation 30:

$$J_z(\text{no wedge}) \;\approx\; -\frac{\left(\frac{2}{\beta_v\gamma} \;-\; \frac{1}{(\beta_v\gamma)^3}\; ln(A\;\beta_v^4\gamma^4) \;+\; \frac{2}{(\beta_v\gamma)^3}\right)}{\left(\frac{1}{2}\; ln(A\;\beta_v^4\gamma^4 \;-\; \beta_v^2\right)}\;\beta_v^3\gamma \tag{33}$$

It is seen that J_z is strongly negative at low energies (longitudinal heating), and is only barely positive at momenta above 300 MeV/c. In practice there are many reasons to cool at a moderate momentum around 250 MeV/c, where $J_z \;\approx\; 0$. However, the 6D cooling is still strong $J_6 \approx 2$.



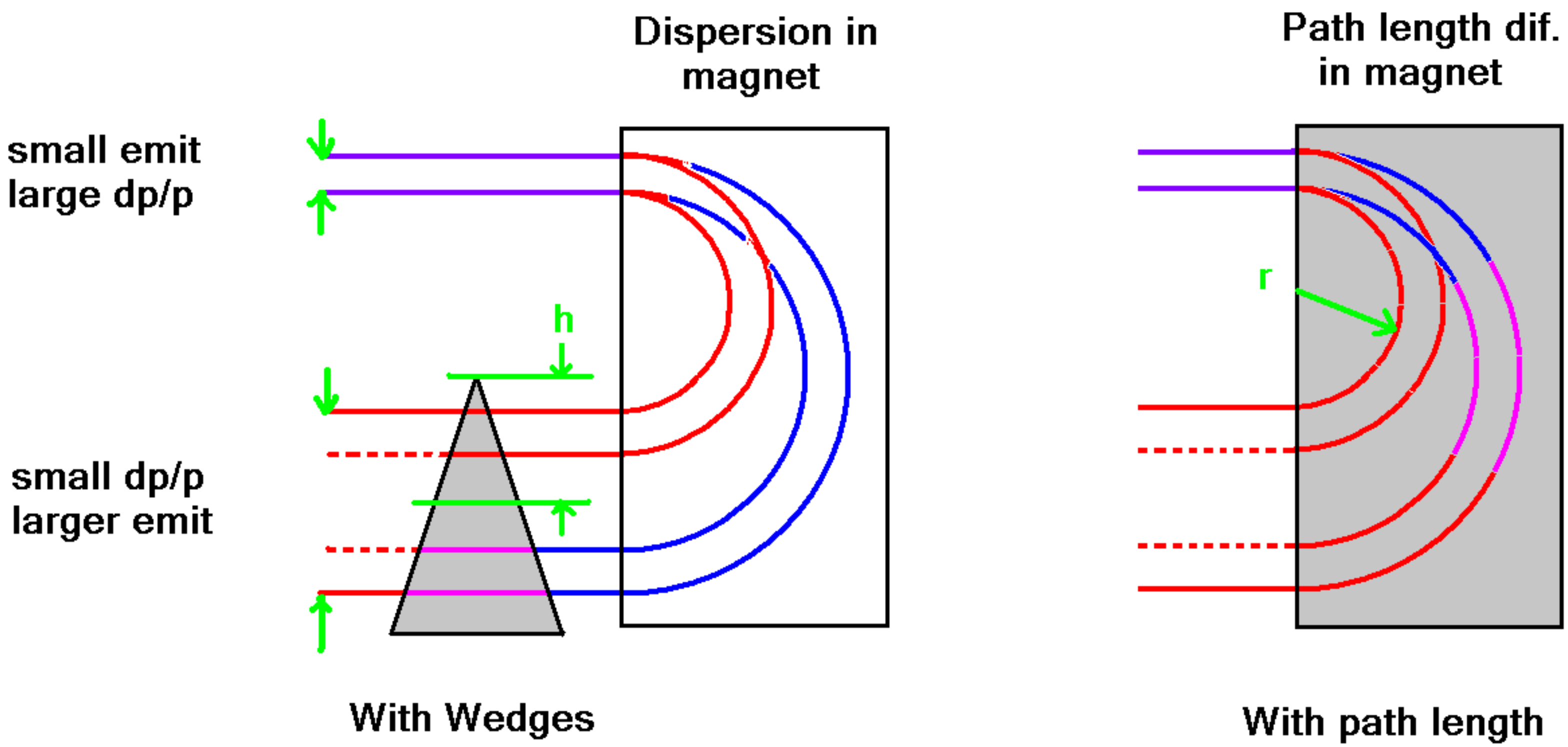
What is needed is a method to exchange cooling between the transverse and longitudinal direction s. This is done in synchrotron cooling if focusing and bending is combined, but in this case, and in general, one can show that such mixing can only increase one J at the expense of the others: J_6 is conserved.

$$\Delta J_x + J_x + J_x = 0 \tag{34}$$

and for typical operating momenta:

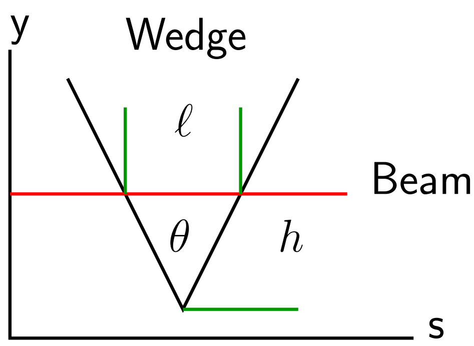
$$J_x \;+\; J_y \;+\; J_z \;=\; J_6 \;\approx\; 2.0 \tag{35}$$

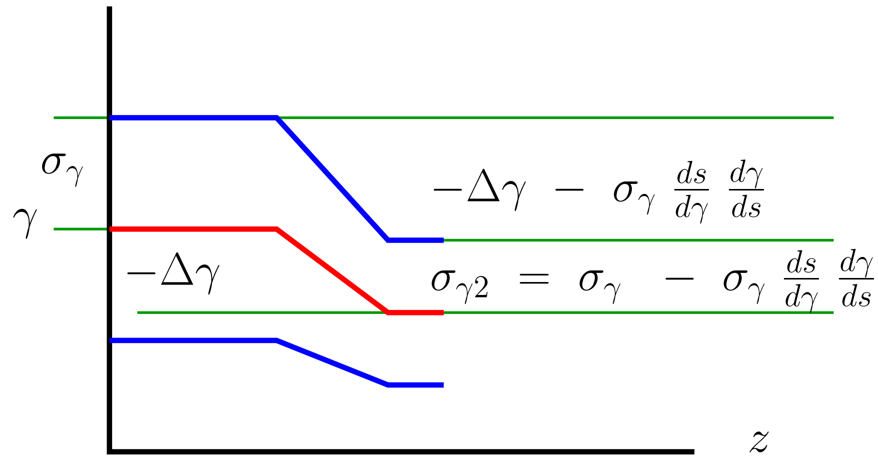
3.3 Emittance Exchange



- dp/p reduced But σ_y increased
- Long Emittance reduced Trans Emittance Increased
- "Emittance Exchange"

3.4 Longitudinal cooling with wedges and Dispersion





For a wedge with center thickness ℓ and height from center h ($2h\ \tan(\theta/2) = \ell$), in dispersion D ($D = \frac{dy}{dp/p}$, or with eq.2: $D = \beta_v^2\ \frac{dy}{d\gamma/\gamma}$) (see fig. above):

$$\frac{\Delta\epsilon_z}{\epsilon_z} \;=\; \frac{\Delta\sigma_\gamma}{\sigma_\gamma} \;=\; \frac{\sigma_\gamma\ \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right)}{\sigma_\gamma} \;=\; \frac{ds}{d\gamma}\left(\frac{d\gamma}{ds}\right) \;=\; \left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\ \left(\frac{d\gamma}{ds}\right)$$

and

$$\frac{\Delta p}{p} \;=\; \frac{\Delta\gamma}{\beta_v^2\gamma} \;=\; \frac{\ell}{\beta_v^2\gamma}\ \left(\frac{d\gamma}{ds}\right)$$

So from the definition of the partition function J_z:

$$\Delta J_z(\text{wedge}) \;=\; \frac{\frac{\Delta\epsilon_z}{\epsilon_z}}{\frac{\Delta p}{p}} \;=\; \frac{\left(\frac{\ell}{h}\right)\frac{D}{\beta_v^2\ \gamma}\ \left(\frac{d\gamma}{ds}\right)}{\frac{\ell}{\beta_v^2\gamma}\ \left(\frac{d\gamma}{ds}\right)} \;=\; \frac{D}{h} \qquad (\text{for simple bend \& gas } \Delta J_z(\text{wedge}) = 1) \quad (36)$$

$$J_z \;=\; J_z(\text{no wedge}) \;+\; \Delta J_z(\text{wedge}) \qquad (37)$$

But from eq.34, for any finite $J_z(\text{wedge})$, J_x or J_y will change in the opposite direction.

3.5 Longitudinal Heating Terms

Since $\epsilon_z = \sigma_\gamma \, \sigma_t \, c$, and t and thus σ_t is conserved in an interaction

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{\Delta\sigma_\gamma}{\sigma_\gamma}$$

Straggling, from Perkins text book, converted to MKS:

$$\Delta(\sigma_\gamma) = \frac{\Delta\sigma_\gamma^2}{2\sigma_\gamma} \approx \frac{1}{2\sigma_\gamma} \, 0.06 \, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho \, \Delta s$$

From eq. 2: $\Delta E = E\beta_v^2 \, \frac{\Delta p}{p}$, so:

$$\Delta s = \frac{\Delta E}{dE/ds} = \frac{1}{dE/ds} E \, \beta_v^2 \, \frac{\Delta p}{p}$$

so

$$\frac{\Delta\epsilon_z}{\epsilon_z} = \frac{0.06}{2\sigma_\gamma^2} \, \frac{Z}{A} \left(\frac{m_e}{m_\mu}\right)^2 \gamma^2 \left(1 - \frac{\beta_v^2}{2}\right) \rho \, \frac{\beta_v^2 \, E}{dE/ds} \, \frac{\Delta p}{p}$$

This can be compared with the cooling term

$$\frac{\Delta\epsilon_z}{\epsilon_z} = - \, J_z \, \frac{dp}{p}$$

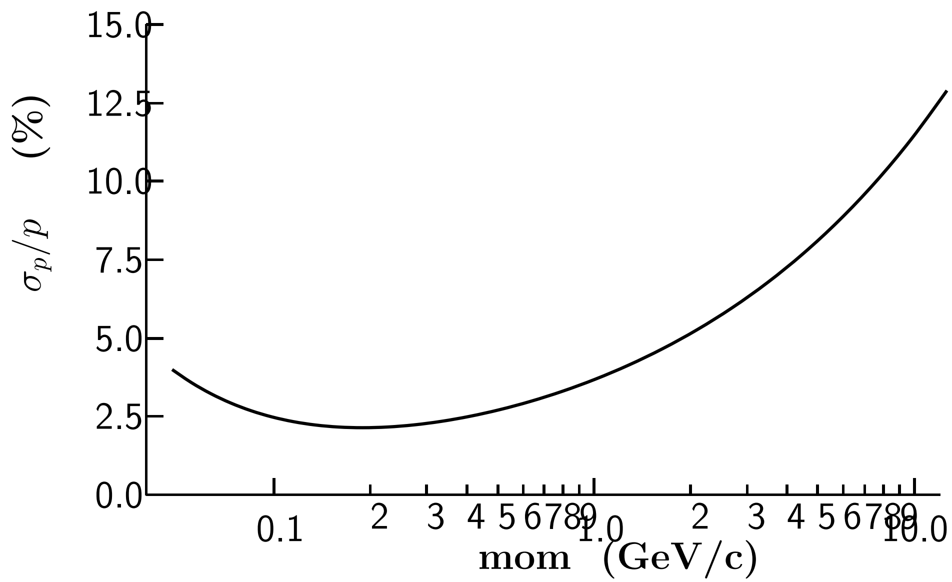
giving an equilibrium:

$$\frac{\sigma_p}{p} = \left(\left(\frac{m_e}{m_\mu}\right) \sqrt{\frac{0.06 \; Z \; \rho}{2 \; A \; (d\gamma/ds)}}\right) \sqrt{\frac{\gamma}{\beta_v^2} \left(1 - \frac{\beta_v^2}{2}\right) \quad \frac{1}{J_z}} \tag{38}$$

For Hydrogen, the value of the first parenthesis is ≈1.36 %.

Without coupling, J_z is small or negative, and the equilibrium does not exist. But with equal partition functions giving $J_z \approx 2/3$ then this expression, for hydrogen, gives: the values plotted below.

The following plot shows the dependency for hydrogen



It is seen to favor cooling at around 200 MeV/c, but has a broad minimum.

3.5.1 rf and bunch length

To obtain the Longitudinal emittance we need σ_z.

If the rf acceleration is relatively uniform along the lattice, then we can write the synchrotron wavelength[2] :

$$\lambda_s \;=\; \sqrt{\frac{2\pi\beta_v^2\lambda_{rf}\gamma\;\;[mc^2/e]_\mu)}{\mathcal{E}_{\mathrm{rf}}\alpha\,\cos(\phi)}} \tag{39}$$

where, in a linear lattice, the "momentum compaction" is:

$$\alpha \;=\; \frac{\frac{dv_z}{v_z}}{\frac{dp}{p}} \;=\; \frac{1}{\gamma^2} \tag{40}$$

and the field $\mathcal{E}_{\mathrm{rf}}$ is the rf accelerating field; ϕ is the rf phase, defined so that for $\phi = 0$ there is zero acceleration.

The bunch length, given the relative momentum spread $dp/p = \delta$, is given by[3]:

$$\sigma_z \;=\; \delta\;\beta_v\frac{\alpha\;\lambda_s}{2\pi} \;=\; \delta\;\beta_v^2\;\sqrt{\frac{\lambda_{rf}\;[mc^2/e]_\mu}{2\pi\;\gamma\;\mathcal{E}\;\;\cos(\phi)}} \tag{41}$$

This, in the following plot, is seen to be only weakly dependent on the energy, but the longitudinal emittance $\epsilon_z \;=\; \beta_v\gamma\;\sigma p/p\;\sigma_z$ rises almost linearly with momentum, strongly favoring low momenta.

It is also apparent that the emittance can be reduced if a higher frequency and higher gradient rf is used. The limit her is when the ratio of σ_z/λ becomes too large and particles do not remain in the bucket.

[2] e.g. s y Lee "Accelerator Physics", eq 3.27
[3] e.g. s y Lee "Accelerator Physics", eq 3.55

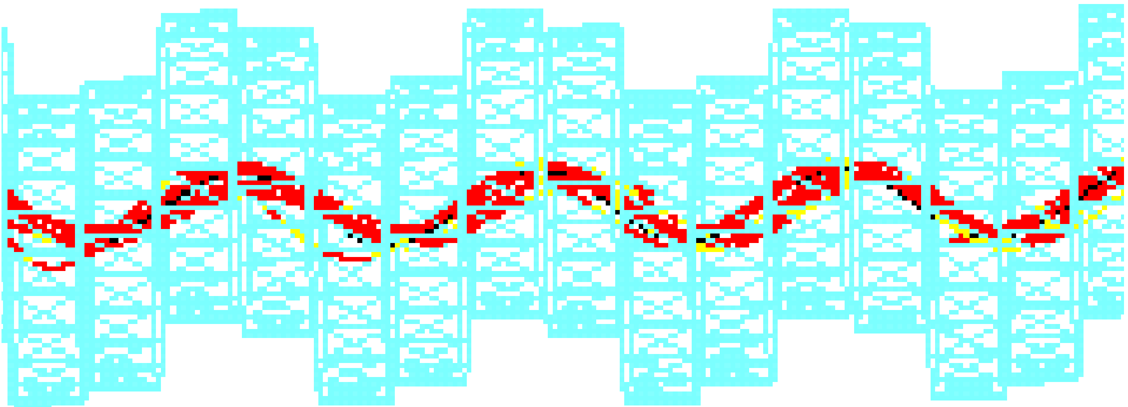
3.6 Emittance Exchange Studies

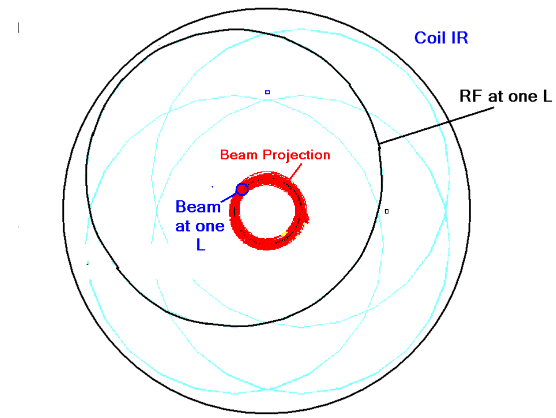
- Wedges in Bent Solenoids
  Problem with rf due to slip
- Ring with with solenoid focus (Balbakov[4])
  achieved Merit=90 but not real fields
- Quadrupole focused ring (Garren et al [5])
  achieved Merit ≈15 , no end fields
- Ring with Maxwellian Bend only focusing (Garren et al)
  achieved Merit 10-100
- RFOFO Ring (Palmer et al[6])
  achieved Merit ≈140 with real fields
- Wedges in with Maxwellian Helical fields[7]
  Good performance, but fields very high if coils outside rf

---

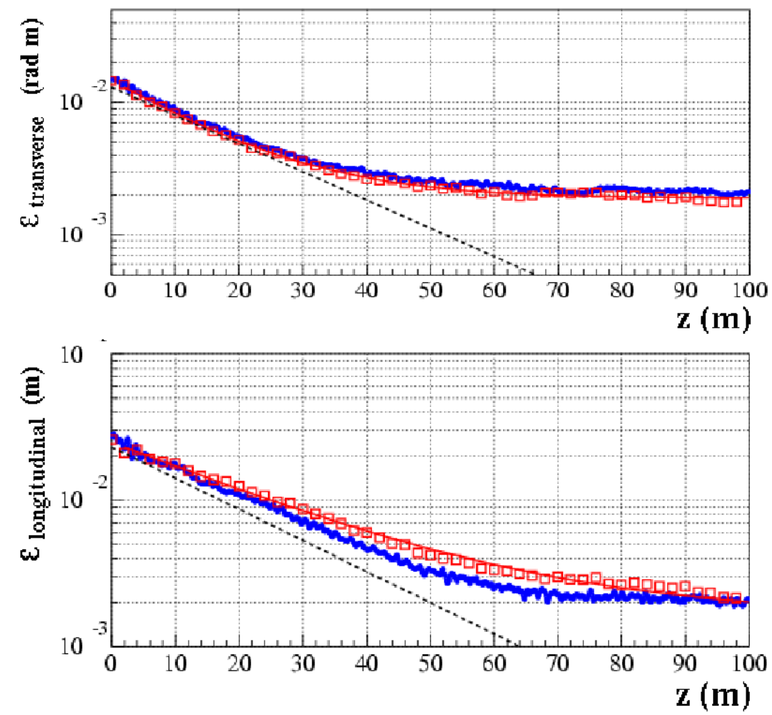
4 MUC-232 & 246
5 Snowmass Proc.
6 MUC-239
7 MUC-146, 147, 185, 187, 193, **284**

3.7 Example 1) In Gas with Helical Field





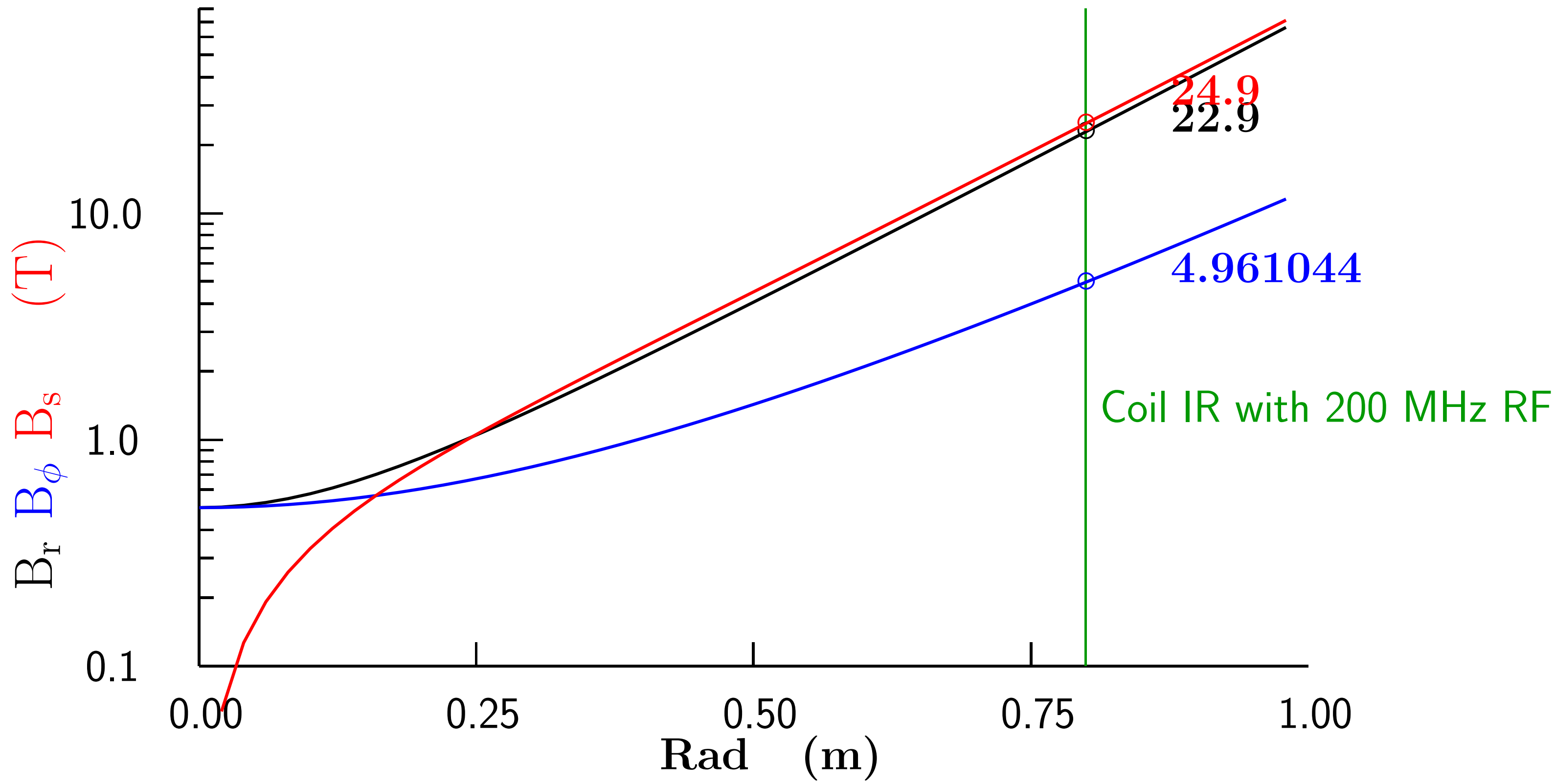
- Gas used partly for higher gradients
  Not yet demonstrated
- $J_z < 1$ can be set to 2/3
- Cooling in 6 dimensions
  of order 1000
- Moderate fields at beam
  Bz=3.5 T. Br=.5 T
- Better Performance than
  RFOFO Ring



[8] MUC 185 and 284

But Helix Fields at Coils > 24 T

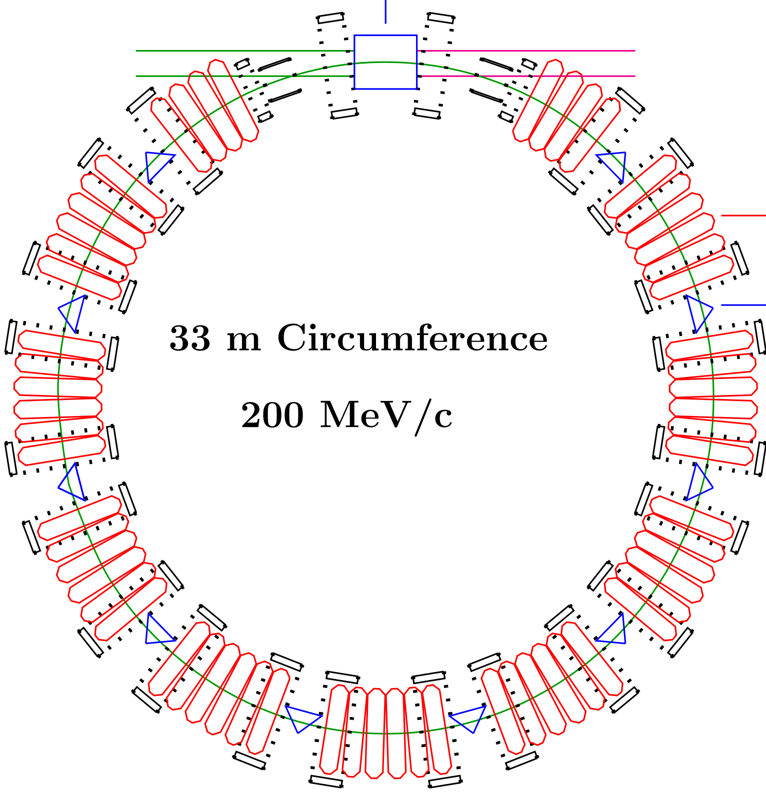
For: $\lambda = 1$ m
$B_\perp = 0.5$ T



- Increasing pitch: hurts ds/dp
- Decreasing helix B: hurts ds/dp
- Lowering RF λ $\rightarrow$ lower emit + higher B's
- Exploring emittance exchange before bunching and RF

3.8 Example 2) RFOFO Ring

R.B. Palmer R. Fernow J. Gallardo[9], and Balbekov[10]



[9] Fernow and others: MUC-232, 265, 268, & 273
[10] V.Balbekov "Simulation of RFOFO Ring Cooler with Tilted Solenoids" MUC-CONF–0264

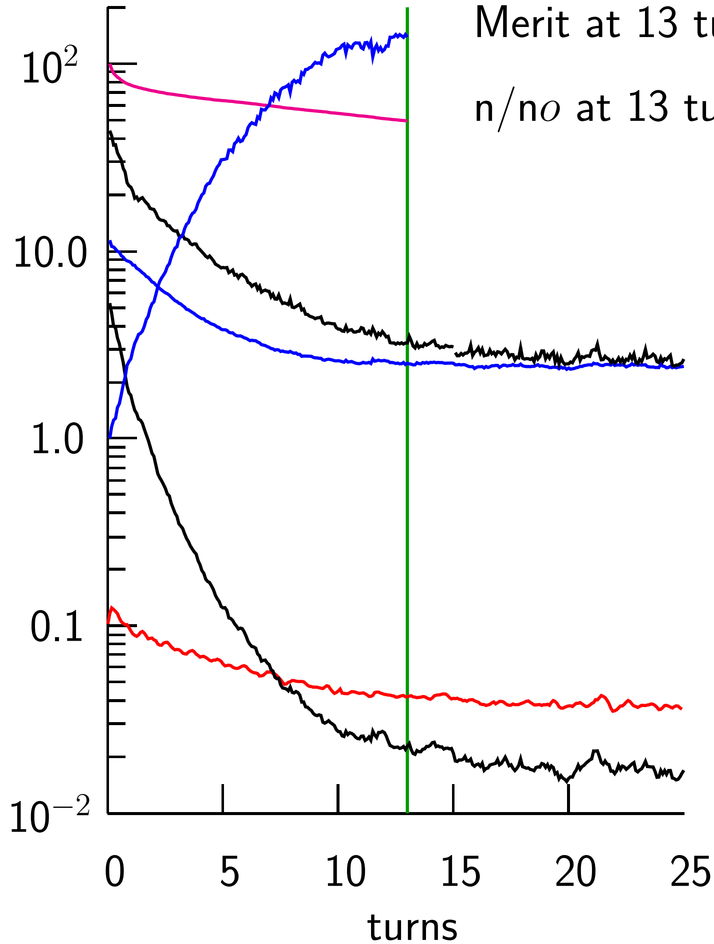
(

Performance

Using Real Fields, but no windows or injection insertion

$$\text{Merit} = \frac{n}{n_o} \frac{\epsilon_{6,o}}{\epsilon_6} = \frac{\text{Initial phase density}}{\text{final phase density}}$$

$n/n_o =$ 1543 / 4494



Merit at 13 turns 139 Falls after 13 turns from decay loss

n/no at 13 turns 0.50

$\epsilon \parallel$ 43.9 to 2.65 ($\pi\ mm$)
$\epsilon \perp$ 11.4 to 2.43 ($\pi\ mm$)

dp/p 10.2 to 3.6 %
$\epsilon 6$ 5.3 to 0.017 $(\pi\ mm)^3$

3.8.1 Compare Simulation with theory

$D = 7$ cm, $\ell = 28.6$ cm, and

$$h = \frac{\ell}{2 \ \tan(100^o/2)} = 12 \quad \mathrm{cm}$$

$$J_z = \frac{D}{h} = 0.58$$

Since there is good mixing between x and y so $J_x = J_y$, and from equ 35, $\Sigma J_i \approx 2.0$, so

$$J_x = J_y \approx \frac{2 - 0.58}{2} = 0.71$$

i.e. The wedge angle gave nearly equal partition functions in all 3 coordinates, and gives the maximum merit factor.

The theoretical equilibrium emittances are now (eq.19):

$$\epsilon_\perp(\mathrm{min}) = \frac{C \ \beta_\perp}{J \ \beta_v} = \frac{38 \ 10^{-4} \ 0.4}{0.71 \ 0.85} = 2.5 \ \ (\pi \ mm)$$

c.f. 2.43 (π mm) observed, which is very good agreement considering the approximations used.

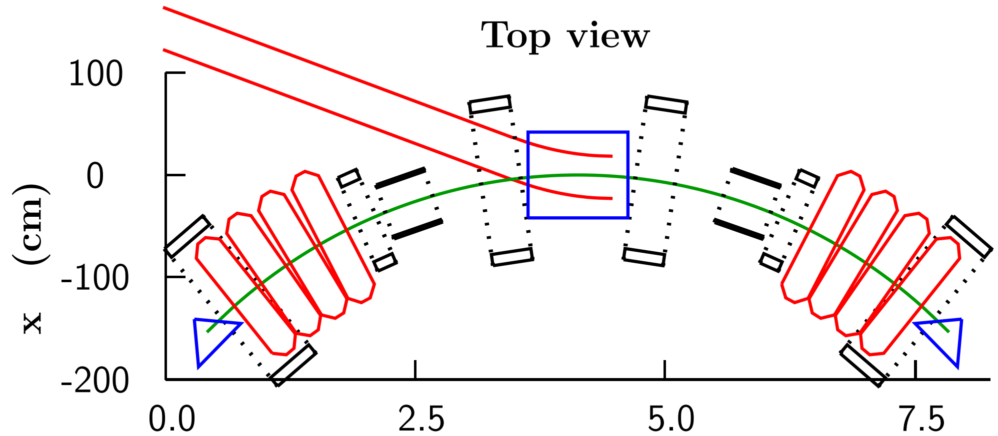
And from equation 38 we expect

$$\frac{dp}{p}(\mathrm{min}) \ \approx 2.3\%$$

compared with 3.6% observed, which is less good agreement. This may arise from the poorer approximation of the real Landau scattering distribution by a simple Gaussian.

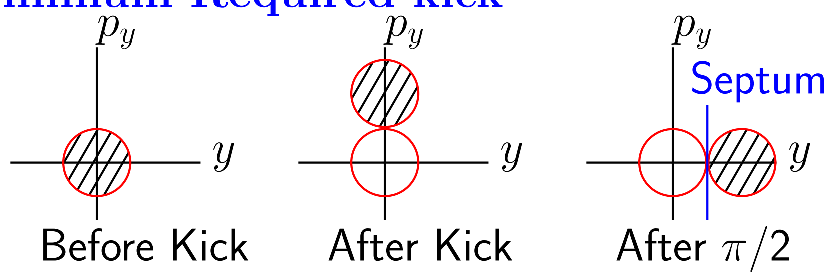
3.8.2 Insertion for Injection/Extraction

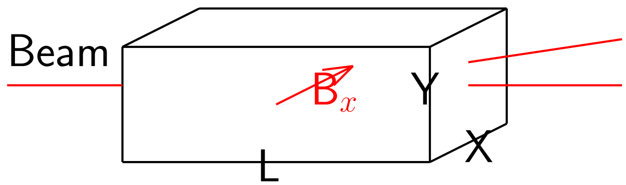
.



- Merit Factor 139 → 100

Minimum Required kick





$$f_\sigma = \frac{\mathrm{Ap}}{\sigma} \qquad \mu\ (\text{of return}) = \inf \qquad F = \frac{Y}{X}$$

$$I = F \left(\frac{4\ f_\sigma^2\ [m_\mu\ c^2/e]}{\mu_o\ c} \right) \quad \frac{\epsilon_n}{L}$$

$$V = \left(\frac{4\ f_\sigma^2\ [m_\mu\ c^2/e]\ R}{c} \right) \quad \frac{\epsilon_n}{\tau}$$

$$U = F \left(\frac{[m_\mu\ c^2/e]^2\ 8\ f_\sigma^4\ R}{\mu_o\ c^2} \right) \quad \frac{\epsilon_n^2}{L}$$

- muon ϵ_n $\gg$ other ϵ_n's
- So muon kicker Joules $\gg$ other kickers
- Nearest are $\bar{p}$ kickers

Compare with others

For $\epsilon_\perp = 10\ \pi$mm, (Acceptance=90 pi mm) $\beta_\perp = 1$m, & τ=50 nsec:
After correction for finite μ and leakage flux:

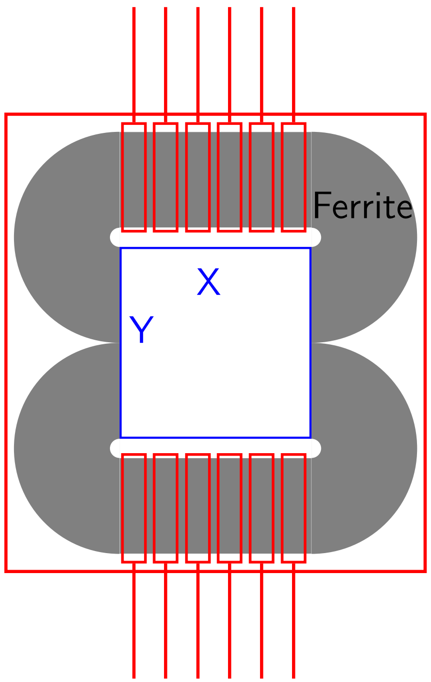
		μ Cooling	CERN $\bar{p}$	Ind Linac
$\int B d\ell$	Tm	.30	.088	
L	m	1.0	≈5	5.0
$t_{\rm rise}$	ns	**50**	**90**	**40**
B	T	**.30**	**≈0.018**	**0.6**
X	m	.42	.08	
Y	m	.63	.25	
$V_{\rm 1turn}$	kV	**3,970**	**800**	**5,000**
$U_{\rm magnetic}$	J	**10,450**	**≈13**	**8000**

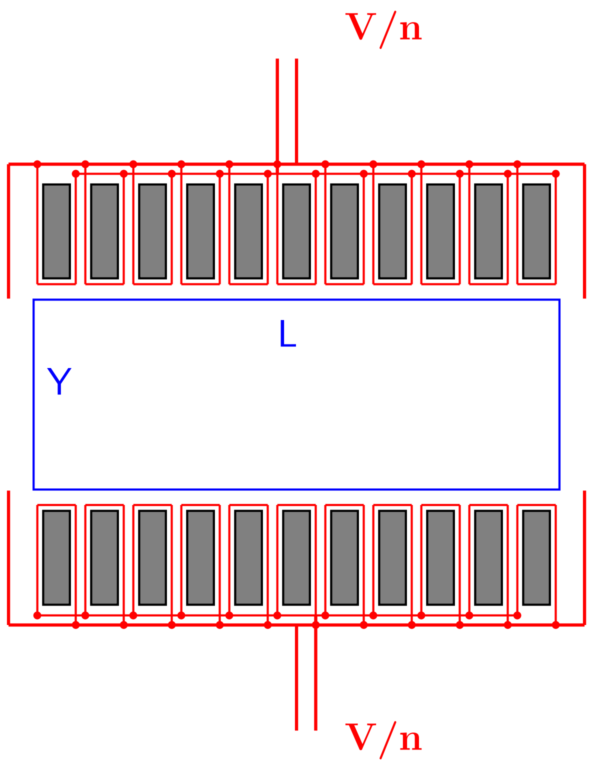
Note

- U is 3 orders above $\bar{p}$, and 1 order of magnitude more than 30 pi mm FFAG
- Same order as Induction
- And t same order as a few m of induction linac
- But V is too High for single turn kicker

Induction Kicker

- Drive Flux Return
- Subdivide Flux Return Loops
  Solves Voltage Problem
- Conducting Box Removes
  Stray Field Return

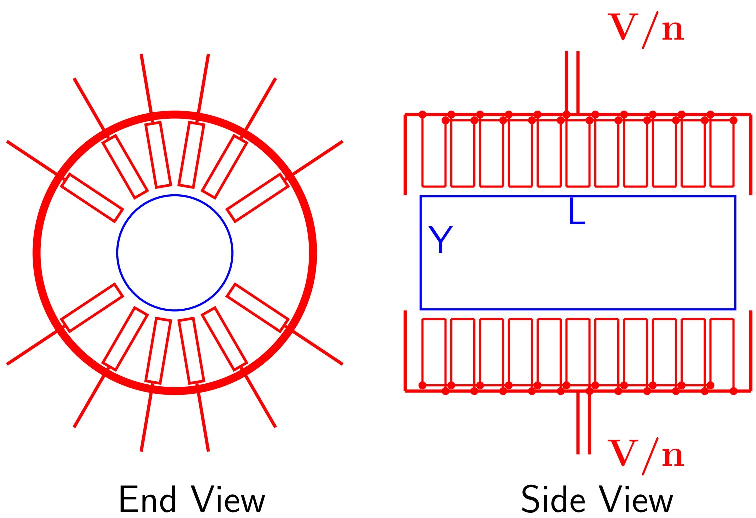




End View Side View

Works with no Ferrite

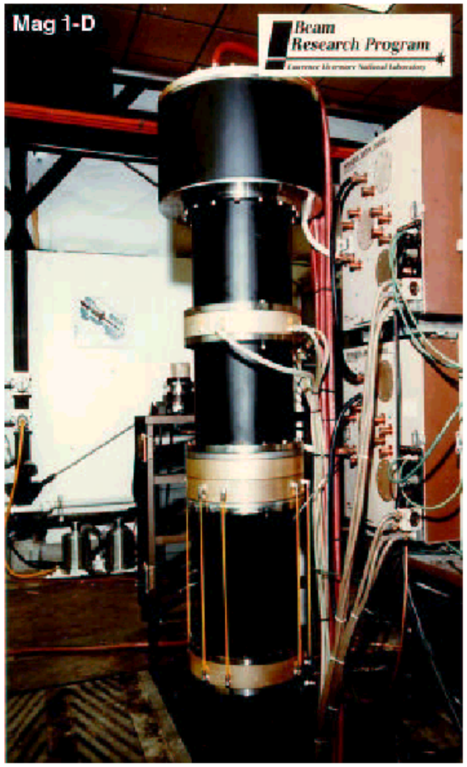
- V = the same
- U≈ $2.25\times$
- I≈ $2.25\times$
- No rise time limit
- Not effected by solenoid fields

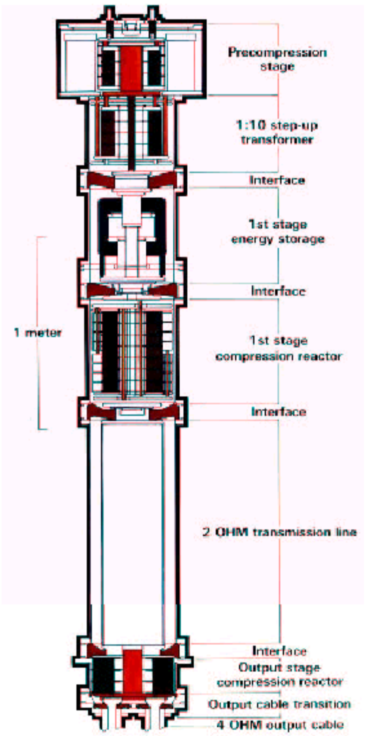


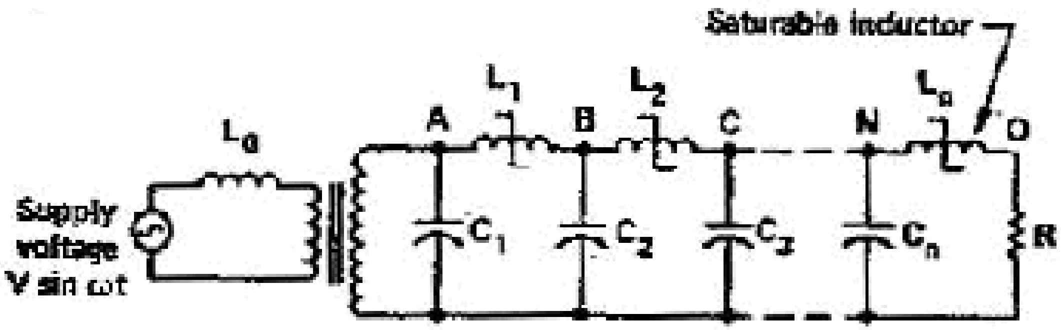
- If non Resonant: 2 Drivers
  for inj. & extract.
  Need 24 $\times$2 Magamps (≈ 20 M$)
- If Resonant: 1 Driver, 2$\times$efficient
  Need 12 Magamps (≈ 5 M$)

Magnetic Amplifiers

Used to drive Induction Linacs similar to ATA or DARHT







3.9 Longitudinal Cooling Conclusion

- Good cooling in 6 D in a ring
  - But injection/extraction difficult
  - Requires short bunch train
- Good 6D cooling in Gas Helix
  - But required very high fields at coils outside RF
- Converting Ring cooler to a large Helix
  - Solves Injection/extraction problem
  - Solves bunch train length problem
  - Allows tapering to improve performance
  - But more expensive than ring
  - Needs more study

4 TUTORIALS

Files are in

http://pubweb.bnl.gov/people/palmer/04school/icool2z/ make a new comand line directory and copy all these files into it.

These files include an icool executable, a basic compiler, a topdraw plotter.

You may later want to use your own compilers and plotters, but this way we can hopefully get instant results.

===

Try typing any of the following: any one should execute and give a plot on the screen

page down should show more plots

- runtrack focus
- runtrack focus0
- runtrack focus1
- runtrack focus2
- runbeta fs2
- runlong cont (but not yet)
- runring ring (nor this yet)

4.1 Introduction

All our ICOOL jobs read files: for001.dat (data) and for003.dat (input tracks) , and for020.dat (coild desciption) or for045.dat (field description).

They will write for002.dat (a log file) and and for009.dat (an ntuple) among others.

I have short basic programs to read the ntuple and generate top draw plot files: ###.td which can be converted into tex files for printing.

To keep track of these files when running different jobs, it is convenient to save them with a job name that I will write as ###. The files are then kept with names:###.coi ###.f01, ###.f03 etc

4.2 Two Batch Commands: "runtrack", "new"

Command to Run Program

Type: "runtrack ###"   e.g. "runtrack focus"

this executes the following batch job (runtrack.bat)

copy %1.f01 for001.dat   copy main data file
copy %1.f03 for003.dat   copy input tracks
copy %1.coi dirty.dat   copy coil definitions
cleaning   remove comments after !'s
copy clean.dat coil.dat   copy cleaned up coil file
sheet3   Make multiple current sheets for coil blocks
copy sheet.out for020.dat   copy sheet data
icool   Run ICOOL
TRACK2   Run analysis of ntuple file to make plots
copy coil.td + track.td %1.td Copy plot files
VU %1.td   Vue plots with TOPDRAW

A Usefull batch command: new ##1 ##2

copy the "set" of files to a new name prior to making modifications
e.g. use: "new focus2 newf1"
names may not be more than 8 characters

```
COPY %1.F01 %2.F01
COPY %1.F03 %2.F03
COPY %1.F45 %2.F45
copy %1.coi %2.coi
```

4.3 Example 1, a very simple case: "focus"

Main data file: focus.f01

this will be copied to for001.dat main data used by icool

```
 Drift space example                           ! a title
 $cont npart=1                                 ! no of tracks =1
 nprnt=3 prlevel=1 bgen=.false. $
 $ints $
 $nhs  $
 $nsc  $
 $nzh nzhist=0  $                              ! no of crude plots vs z
 $nrh  $
 $nem  $
 $ncv  $
SECTION
REPEAT                                         ! repeat till "ENDREPEAT"
150                                            ! 150 times
OUTPUT                                         ! write out data to for009.dat
SREGION                                        ! start an 8 line "region"
0.05   1  0.001                                ! deltaz 1 zstep   (m)
1  0.  0.10                                    ! 1 0 rmax
SOL                                            ! solenoid
1 0 0 0. 1.8 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.  ! 1 0 0 0 Bz 0 0 0 0 0 0 0 0 0 0
VAC                                            ! no material, could be CU, BE etc
CBLOCK                                         ! dummy material shape
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.             ! dummy material shape

ENDREPEAT                                      ! end repeat

ENDSECTION                                     ! end of run
```

Input tracks file: ###.f03

###.f03 copied to for003.dat initial tracks used by icool

this example (focus.f03) has only two tracks. Add one line each for more.

```
focus                                     ! title
0  0  0  0  0  0  0  0                    ! used for restarting, ignore here
1 0 2 0 0 1 0 0 0 .005 .005 .2 0 0 1     !i 0 mu 0 t wt x y z px py pz Px Py Pz
2 0 2 0 0 1 0 0 0 .01  .01  .2 0 0 1     !i 0 mu 0 t wt x y z px py pz Px Py Pz
```

lengths in m, momenta in GeV/c, t in seconds. P's are polarization

Coil File ###.coi

In this case the field is defined in the ###.f01 file so the coil file is ignored

Log file: FOR002.dat

for002.dat log written by icool which lists of regions, error messages, and crude plots (I do not use these)

Ntuple output file: FOR009.dat

written by the "OUTPUT" commands in the for001.dat data file
The first line hs a title, the second units, then the track data. e.g.

```
# Drift space example                          ! title
#  units = [s] [m]  [GeV/c] [T] [V/m]          ! units
  i par typ flg reg t x y z Px
```

4.4 Example 2: .f01 Example with coil Specified

```
 focus1                                   ! title
 $cont npart=1                            ! number of particles to track
 nprnt=3 prlevel=1 bgen=.false. $         !
 $ints $                                  !
 $nhs  $                                  !
 $nsc  $                                  !
 $nzh nzhist=0  $                         ! no of crude plots vs z
 $nsc  $                                  !
 $nzh  $                                  !
 $nrh  $                                  !
 $nem  $                                  !
 $ncv  $                                  !
SECTION                                   ! start
CELL                                      ! cell over which sheet fields apply
1                                         ! number of identical cells
.true.

SHEET                                     ! fields from sheets in for020.dat
3 20 .0025 .0025 6 0.24 99 1 0 0 0 0 0 0 0 ! mode file dl dr l r reach interp

REPEAT                                    ! repeat following regions
120                                       ! 120 times
OUTPUT                                    ! print ntpl here
SREGION                                   ! a region has 8 lines
0.05   1  0.001                           ! deltaz 1 zstep
1  0.   0.24                              ! 1 0 rmax
NONE                                      ! dummy for RF or other field
  0. 0. 0 0 0 0 0 0 0 0 0 0 0 0 0         !
VAC                                       ! vacuum
CBLOCK                                    ! dummy for shape of material
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.
ENDREPEAT                                 ! end repeat loop
ENDCELL                                   ! end cell
ENDSECTION                                ! end of everything
```

Coil Definitions ###.coi

for020.dat coil sheets used by icool is generated by basic prog SHEET3 using coil descriptions in ###.coi

e.g. focus2.coi

```
alternating strong sols new
0 1  1. 1. 1 -.001  10    !zstart nrepeat zfac rfac Ifac z1 z2
2   .25  300000  .5  1 .05 3        !gap r1 I len 1 dr nsheets
.5   .25  -200000  .5  1 .03 3        !gap r1 I len 1 dr nsheets
0 0 0 0 0 0 0                     !end data on coils
0 1 1 1 1                         !zstart nrepeat zfac rfac Ifac
0 0 0 0                           !end data on picture
```

which generates the following "for020.dat" format and a topdraw picture in "coil.td"

```
alternating strong sols new
 6  1
 1  2  .5  .2583333  600000
 2  2  .5  .275  600000
 3  2  .5  .2916667  600000
 4  3  .5  .255 -400000
 5  3  .5  .265 -400000
 6  3  .5  .275 -400000
```

in this case 3 sheets for each coil specified in the .coi

4.5 An example with material for cooling: "cont"

"cont.f01"

```
C1 Continuous cooling
 $cont npart=100 nsections=1 timelim=500. bgen=.false.
 varstep=.true. nprnt=1 prlevel=-1 epsstep=1e-4 ntuple=.false.
 phasemodel=3 neighbor=.false. dectrk=.true.
 fsav=.false. izfile=1160  bunchcut=1. spin=.true. output1=.true.
 timelim=9999 $
 $bmt nbeamtyp=1 $
1 3 1. 1                                      ! 2ndary pion---not used because bgen=false above
0.     0.0      0.179      0.      0.      0.200  !mean: x y z px py pz
0.     0.       0.         0.0   0.0   0.         !sigs
0
 $ints ldecay=.true.  declev=1            !details of scattering and straggling  - see manual
ldedx=.true. lstrag=.true. lscatter=.true.
  delev=2 straglev=4 scatlev=4  $
 $nhs  $                                  !
 $nsc  $                                  !
 $nzh nzhist=0  $                         ! no of crude plots vs z
 $nsc  $                                  !
 $nzh  $                                  !
 $nrh  $                                  !
 $nem  $                                  !
 $ncv  $
SECTION
REFP
2   .2   0.   0  3                            !muon  reference-momentum   0 0 3
BEGS
CELL
50                                            ! number of following cells
.true.                                        ! alternating signs of Bz in each cell

SHEET              !  ========================= cool t53
3.  20  0.01  0.01  2.80  0.4  10.      2. 99. 0.  0. 0. 0. 0. 0.
SREGION          ! 1/2 HYDROGEN
0.175      1    3e-3
```

```
1  0.  0.18
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
LH
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

SREGION         !  Hydrogen window
0.0025      1    2e-3
1  0.   0.5
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0.  0. 0. 0. 0. 0.
AL
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

SREGION         ! 1st  free
0.2575      1    2e-3
1  0.   0.5
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0.  0. 0. 0. 0. 0.
VAC
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

REPEAT
3
SREGION         !RF
0.470  1    5e-3
1  0.   0.65
ACCEL
2.  201.25   15.48  29.80  0.   0. 0. 0. 0. 0.   0. 0. 0. 0. 0. ! mode freq
VAC
NONE
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

ENDR
```

```
SREGION         ! RF 4
0.47  1    5e-3
1  0.   0.65
ACCEL
2.  201.25   15.48  29.80  0.   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.
VAC
NONE
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

SREGION         !  free
0.2575       1  2e-3
1  0.    0.5
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0.  0. 0. 0. 0. 0.
VAC
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

SREGION         !  Hydrogen window
0.0025      1    2e-3
1  0.   0.5
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0.  0. 0. 0. 0. 0.
AL
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.

OUTPUT
SREGION         ! 2nd 1/2 absorber
0.175      1    3e-3
1  0.  0.18
NONE
  0. 0. 0. 0. 0.   0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
LH
CBLOCK
   0. 0. 0. 0. 0.   0. 0. 0. 0. 0.
ENDCELL
ENDSECTION
```

4.6 Description of Jobs

1. runtrack focus fixed field
2. runtrack focus0 Long focus coil
3. runtrack focus1 single short focus coil
4. runtrack focus2 two focus coils
5. runbeta fs2 get betas vs mom for Study 2 Lattice
6. runlong cont cool in a long Study 2 lattice
7. (runring ring cool in a ring)

4.7 Excercise 1

1. Run the focus examples by typing:
   - "runtrack focus" fixed field
   - "runtrack focus0" Long focus coil
   - "runtrack focus1" single short focus coil
   - "runtrack focus2" two focus coils

2. make a new file called test1 from "focus1" using "new". Modify the new file to explore sensitivity to initial angles.

   in test1.f01: Note the max radius in the region command and in the sheet command that sets up the field grid.

3. In test1.coi: Move the start of the coil to 0.5 m (instead of 3m)

4. In test1.coi: Increase the current so the beam is foucused near the end of z

5. In test1.f03: Add further tracks with increased initial pt till the tracks pass outside the radius limits.

   Do they all focus to the same point?

   What is the name for this abberation?

   Is it positive or negative?

4.8 Excercise 2: Determine betas of a lattice

1. type "runbeta fs2"
2. using "new": make a new file from "fs2" called "beta1" .

   Then In beta1.coi::

   a) Increase the two "focus" coil currents by approx 20% which will give smaller betas

   b) while decreasing the single "coupling" coil current to obtain betas more or less centered on 0.2 GeV/c

   How much was the center beta reduced?

   Is the momentum acceptance the same?
3. Repeat the above calling it "beta2" with the two "focus" coils currents from fs2 by approx 40%
4. Repeat the above calling it "beta3" with the two "focus" coils currents from fs2 by exactly 66%

   what is special about 66%?

4.9 Excercise 3: Cooling in a long lattice

1. run "runlong cont"
2. increase the number of particles to 1000 (on line 3 of cont.f01) and run when you have the time to wait.
3. Make a new file set and then substitute a .coi from the previous excersise that had a smaller beta.

   Is the final emittance smaller?

   Is the acceptance worse?
4. make new file from "cont" called "LiH".

   replace hydrogen with LiH (LIH) with thickness such as to give same energy loss as H2 (there is a table of dE/dx in the lecture notes). Replace the Al window with Berilium (BE) and run "runlong LiH" with npart=1000.